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Active Vibration Control of Aerospace Structures Using a Modified Positive Position Feedback Method

S. Nima Mahmoodi, Mohammad Rastgaar Aagaah and Mehdi Ahmadian

***Abstract*—A Positive Position Feedback controller is modified and a new active vibration control technique is developed. Unlike the conventional Positive Position Feedback, the new controller separates the damping and stiffness control using two parallel first order and second order compensators. The second order compensator has a damping ratio as low as the damping of flexible structure to provide periodic vibration control. Simultaneously, the high damping is made available through a first order compensator. The new controller is applicable to a strain-based sensing/actuating approach and can be extensively applied to piezoelectrically controlled systems. Control gains are obtained by performing the stability analysis. The controller is verified experimentally using a plate vibration suppression setup. The plate is controlled through two piezoelectric patches and its vibrations are monitored by ten sensors mounted on the surface of the plate. The results confirm that the new controller is able to provide good vibration reduction, with the ability to be used to simultaneously control more than one natural frequency.**

I. Introduction

ACTIVE vibration control of flexible structures by means of smart materials, especially piezoelectric patches, is of interest to many researchers. The application of piezoelectric actuators and shape memory alloys in vibration control is increasing in many areas, from microscale actuators in atomic force microscopes to active vibration control of aircraft bodies [1-2]. Vibrations in aerospace structures appear due to various issues and there are different methods to control the vibration. Using piezoelectric patches to control engine-induced noise and vibration in the passenger cabin of aircraft is an issue that has been studied [3]. Active vibration control has also been used for space structures to control the Solar Array Flight Experiment (SAFE) structure during deployment [4]. Although the vibration control methods used in those research projects worked well, an improved method is required to control the vibrations more efficiently with less control effort.

There are several classical vibration control methods for structures such as lead, velocity feedback and acceleration feedback that depend on the measurement of the displacement, velocity and acceleration, for the feedback [5]. Positive Position Feedback (PPF) control works based on velocity feedback control [6]. PPF is a commonly used control method that is utilized in many applications such as flexible antenna [7], flexible manipulators [8], and spacecrafts [9]. However, in PPF, the control input is applied through a piezoelectric actuator that generates strain instead of direct force. PPF uses a second order compensator in feedback with a large damping value to suppress the vibrations and uses a gain value that is smaller than one to keep the system stable [10]. Sometimes a first order compensator is used in PPF to control the vibration [11], however, this compensator cannot efficiently control the induced periodic vibrations in the system.

This paper provides and experimentally verifies a new technique that is called Modified Positive Position Feedback (MPPF), which is a developed version of PPF. It will be shown that similar to PPF, the new method is unconditionally stable, i.e., the stability is not dependent on the damping of the structure. In PPF, the two parameters of the control gain and damping ratio of the compensator must be chosen carefully in order to provide desirable vibration suppression. Sometimes, system identification is required for choosing the proper control gain and damping ratio for the compensator [12]. PPF can be combined with other methods for more effective control such as an adaptive PPF used for improved control of flexible structures [13]. The method proposed here shows similar abilities. In addition, adaptive control was combined with PPF for controlling vibrations of multi-modes of frequency varying structures [14]. The PPF controller was also considered as an output feedback controller to design an improved control system. The optimal control method was used to develop a control design algorithm to be used in design of PPF controllers [15]. In addition, PPF has been used jointly with delayed feedback for designing more robust controllers [16]. Since both the control gain and damping ratio in PPF are gathered in one compensator in the feedback, they influence each other's performance. Considering this effect, selecting the damping ratio and control gain is important for how well PPF is able to suppress structural vibrations. The proposed method will use two parallel compensators to ease the selection of control gains and provide a better control design.

The MPPF suggested here uses two parallel compensators to eliminate both effects of transient and periodic dynamics of the disturbance. One compensator is a second order filter with very small damping to suppress periodic disturbance.

S.N. Mahmoodi is a visiting assistant professor at Center for Vehicle Systems & Safety at Virginia Tech. and author of correspondence (email: mahmoodi@vt.edu).

M. Rastgaar Aagaah is a postdoctoral research fellow at Center for Vehicle Systems & Safety at Virginia Tech. (email: rastgaar@vt.edu).

M. Ahmadian is a professor and director of Center for Vehicle Systems & Safety at Virginia Tech. (email: ahmadian@vt.edu).

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The other compensator is a first order filter to dissipate transient disturbance. The MPPF controller provides the ability to select proper gains that reduce settling time and controlling effort more efficiently. A plate clamped at four edges will be disturbed by an electromagnetic shaker and two piezoelectric actuators will control the plate vibrations. Two modes will be investigated and controlled using MPPF method.

II. Modified Positive Position Feedback Control

One of the successful control methods for using piezoelectric actuators/sensors as a controlling device is the Positive Position Feedback method. The PPF uses a second order compensator in the feedback of sensor to suppress the vibrations, especially at the resonance frequency. There are three major parameters in this compensator that need to be precisely determined in order to make it efficiently work. The first parameter is the compensator frequency that is usually the same as the resonance frequency if the disturbance excites the resonance frequency. The other two parameters are damping and gain of the compensator. Considering that the compensator frequency is defined then the other two parameters should be carefully selected. Since the objective of the method is suppression of vibration, the damping of the compensator should be large. However, this causes the deduction of the compensator damping frequency in comparison with the system that has small damping and thus the compensator cannot perfectly compensate the vibration of the flexible structure. By decreasing the damping, the settling time will increase. In order to eliminate this disadvantage and improve the efficiency of the controller, a new modified positive position feedback method is proposed that uses two compensators with separate gains. One is a second order compensator with a low damping and the other is a first order compensator parallel with the first one to increase the damping of the closed-loop system.

Here, the general concept of the modified positive position feedback control is introduced. The controller is a resonant controller that uses a collocated control system. It will be shown that it has unconditional stability of the closed-loop system. Such controllers are of interest because of their ability to avoid instability due to spill-over effect [17]. The MPPF controller consists of a second order and a first order controller that work in parallel. The system includes three following equations, the first one describes the structure and the other two describe compensators

$$\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{\Omega}^2\mathbf{x} = \mathbf{C}^T\left(\mathbf{A}\mathbf{y} + \mathbf{B}\mathbf{z}\right) \tag{1a}$$

$$\ddot{\mathbf{y}} + \mathbf{D}_f\dot{\mathbf{y}} + \mathbf{\Omega}_f^2\mathbf{y} = \mathbf{\Omega}_f^2\mathbf{C}\mathbf{x} \tag{1b}$$

$$\dot{\mathbf{z}} + \mathbf{\Omega}_f\mathbf{z} = \mathbf{\Omega}_f\mathbf{C}\mathbf{x} \tag{1c}$$

where $\mathbf{x}$ indicates the $N_m\times 1$ modal vector, $\mathbf{y}$ and $\mathbf{z}$ are the $N_f\times 1$ compensator vectors to suppress N_f modes of the structure, $\mathbf{\Omega}$ is a $N_m\times N_m$ diagonal matrix of modal frequency, $\mathbf{D}$ is a $N_m\times N_m$ diagonal matrix of modal damping, $\mathbf{\Omega}_f$ is a $N_f\times N_f$ diagonal matrix of compensator frequency, $\mathbf{D}_f$ is a $N_f\times N_f$ diagonal matrix of compensator damping whose elements are as small as damping elements of the system, $\mathbf{A}$ and $\mathbf{B}$ are $N_f\times N_f$ diagonal gain matrices with positive elements, and $\mathbf{C}$ is $N_f\times N_m$ participation matrix [14] and is dependent on the modal system.

Theorem 1. The closed-loop system of the structure and compensators of equations (1) are asymptotically stable if and only if $\mathbf{\Omega}^2 - \mathbf{C}^T\left(\mathbf{A}+\mathbf{B}\right)\mathbf{C} > 0$.

Proof. The following nonsingular transformations are made to provide symmetric equations out of equation (1),

$$\mathbf{y} = \mathbf{A}^{-1/2}\mathbf{\Omega}_f\boldsymbol{\psi} \tag{2}$$

$$\mathbf{z} = \mathbf{B}^{-1/2}\mathbf{\Omega}_f^{1/2}\zeta \tag{3}$$

Substituting equations (2) and (3) into (1), the structure and compensators equations can be obtained as:

$$\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{\Omega}^2\mathbf{x} = \mathbf{C}^T\left(\mathbf{A}^{1/2}\mathbf{\Omega}_f\boldsymbol{\psi} + \mathbf{B}^{1/2}\mathbf{\Omega}_f^{1/2}\zeta\right) \tag{4a}$$

$$\ddot{\boldsymbol{\psi}} + \mathbf{D}_f\dot{\boldsymbol{\psi}} + \mathbf{\Omega}_f^2\boldsymbol{\psi} = \mathbf{\Omega}_f\mathbf{A}^{1/2}\mathbf{C}\mathbf{x} \tag{4b}$$

$$\dot{\zeta} + \mathbf{\Omega}_f\zeta = \mathbf{\Omega}_f^{1/2}\mathbf{B}^{1/2}\mathbf{C}\mathbf{x} \tag{4c}$$

The system of equations (4a-4c) is rewritten in the matrix form as:

$$\begin{bmatrix}\ddot{\mathbf{x}}\\ \ddot{\boldsymbol{\psi}}\\ 0\end{bmatrix} + \begin{bmatrix}\mathbf{D} & 0 & 0\\ 0 & \mathbf{D}_f & 0\\ 0 & 0 & \mathbf{I}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{x}}\\ \dot{\boldsymbol{\psi}}\\ \dot{\zeta}\end{bmatrix} + \begin{bmatrix}\mathbf{\Omega}^2 & -\mathbf{C}^T\mathbf{A}^{1/2}\mathbf{\Omega}_f & -\mathbf{C}^T\mathbf{B}^{1/2}\mathbf{\Omega}_f^{1/2}\\ -\mathbf{\Omega}_f\mathbf{A}^{1/2}\mathbf{C} & \mathbf{\Omega}_f^2 & 0\\ -\mathbf{\Omega}_f^{1/2}\mathbf{B}^{1/2}\mathbf{C} & 0 & \mathbf{\Omega}_f\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \boldsymbol{\psi}\\ \zeta\end{bmatrix} = 0 \tag{5}$$

Defining:

$$\boldsymbol{\rho} = \begin{bmatrix}\mathbf{x}\\ \boldsymbol{\psi}\\ \zeta\end{bmatrix};\ \mathbf{L} = \begin{bmatrix}\mathbf{D} & 0 & 0\\ 0 & \mathbf{D}_f & 0\\ 0 & 0 & \mathbf{I}\end{bmatrix}\ \mathit{and}$$

$$\mathbf{P} = \begin{bmatrix}\mathbf{\Omega}^2 & -\mathbf{C}^T\mathbf{A}^{1/2}\mathbf{\Omega}_f & -\mathbf{C}^T\mathbf{B}^{1/2}\mathbf{\Omega}_f^{1/2}\\ -\mathbf{\Omega}_f\mathbf{A}^{1/2}\mathbf{C} & \mathbf{\Omega}_f^2 & 0\\ -\mathbf{\Omega}_f^{1/2}\mathbf{B}^{1/2}\mathbf{C} & 0 & \mathbf{\Omega}_f\end{bmatrix}. \tag{6}$$

The system of equation (5) is asymptotically stable if and only if both $\mathbf{L}$ and $\mathbf{P}$ are positive definite [6]. $\mathbf{L}$ is positive definite since $\mathbf{D}$, $\mathbf{D}_f$ and $\mathbf{\Omega}_f$ are positive definite. In order to evaluate $\mathbf{P}$, it should be proven that for any arbitrary and nonzero vector $\mathbf{k} = [k_1^T \quad k_2^T \quad k_3^T]^T$ the following condition is valid.

$$\mathbf{k}^T\mathbf{P}\mathbf{k} > 0 \tag{7}$$

By expansion of equation (7) and adding and subtracting $k_1\mathbf{C}^T\mathbf{A}\mathbf{C}k_1$ and $k_1\mathbf{C}^T\mathbf{B}\mathbf{C}k_1$, the following condition is obtained,

$$\begin{aligned}&\left(\mathbf{A}^{1/2}\mathbf{C}k_1 - \Omega_f k_2\right)^T\left(\mathbf{A}^{1/2}\mathbf{C}k_1 - \Omega_f k_2\right)\\ &+\left(\mathbf{B}^{1/2}\mathbf{C}k_1 - \Omega_f^{1/2}k_2\right)^T\left(\mathbf{B}^{1/2}\mathbf{C}k_1 - \Omega_f^{1/2}k_2\right)\\ &+k_1^T\left[\Omega^2 - \mathbf{C}^T\left(\mathbf{A}+\mathbf{B}\right)\mathbf{C}\right]k_1 > 0\end{aligned} \tag{8}$$

The first two terms are positive; therefore, the third term must also be positive, hence

$$\mathbf{\Omega}^2 - \mathbf{C}^T(\mathbf{A}+\mathbf{B})\mathbf{C} > 0 \quad (9)$$

For proving the reverse condition, if the condition of equation (9) holds, then the inequality in equation (8) and accordingly equation (7) are correct, meaning **P** is positive definite. Since **L** is positive definite by definition, then the set of equations (5) are stable and accordingly are the system of equations (1).

In order to compare the findings with PPF and provide a better understanding of the gain limits, let's consider the case of vibration control of only the first resonant frequency. To compare the PPF and MPPF stability conditions, the case of vibration control of mode one is considered. Therefore, **C** matrix is only a scalar and the ideal condition would be $\mathbf{C}=[\omega_1]$; then the equation (9) becomes $\omega_1^2-\omega_1^2(\alpha_1+\beta_1)>0$, which can be simplified as $(\alpha_1+\beta_1)<1$. Comparing this with the condition for one mode PPF control ($PPF\ gain<1$), it is realized that the condition of MPPF is similar to PPF, which demands the control gain to be smaller than 1, but in MPPF the summation of both gains must be smaller than 1.

III. Experimental Setup and Method

To validate the proposed modified positive position feedback controller, a major set of experiments are undertaken to generate disturbance on a piezoelectrically active controlled plate and MPPF is used as the active control method. The test rig contains a galvanized steel plate with two piezoelectric patches as control actuators attached to the plate as shown in Fig. 1. The plate is clamped at all edges with a frame. There are 10 accelerometers attached to the plate that are shown with black dots and their corresponding identification number. These numbers will be used further in explaining the results. Two sensors are collocated with the piezoelectric actuators and one is collocated with the point of applying disturbance.

The plate is clamped at all edges with a frame that is bolted to the base by 14 bolts as shown in Fig. 2. The disturbance will be applied using an electromagnetic actuator by Ling Dynamic Systems® that can produce both periodic and impulse disturbances. The disturbance is collocated with the sensor (3) that is on the right side of the piezoelectric patches. This point is slightly offset from the center line to provide different influences on each side of the plate (see Fig. 1); however, it is not that far from center line so the piezoelectric patches that are on the center line can suppress the vibration. There are two piezoelectric patches: one in the center of the plate (PZ 1) and one on the right side of it (PZ 2). Piezoelectric actuators are produced by Piezo Systems, Inc.® (part number T234-A4CL-503X). The mechanical properties of the plate and piezoelectric patches are presented in Table 2.

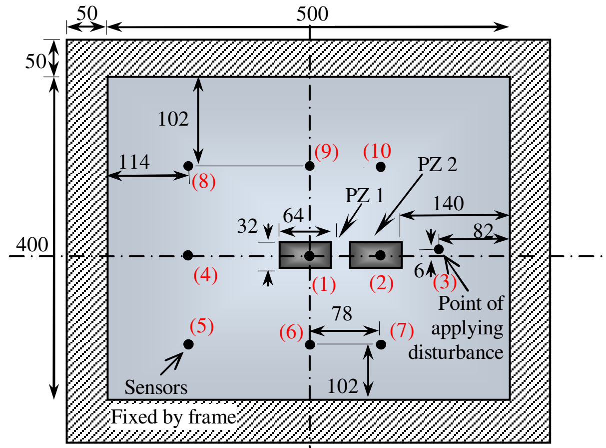

Fig. 1. Schematic of the plate; the unit is millimeter.


Fig 2. Plate and test facilities.

All controlling process and signals are sent and processed via a dSpace® board that uses the ControlDesk® and Matlab Simulink® software packages. In order to identify the resonant frequencies of the plate, a chirp signal is used to excite the system. This will not only reveal the natural frequencies but also identify the resonances with the highest peaks, thus the controller will be used to suppress the vibrations of these modes. Figure 3 shows the frequency response of the plate to the chirp signal that sweeps the frequencies from 0 to 200 Hz.

Table 2. Mechanical properties of plate and piezoelectric material.

Properties	***Value***
Plate modulus of elasticity, E_p (*GPa*)	200±5
Plate density, ρ_p (*kg/m*^{3})	7800
Plate thickness, h_p (*mm*)	0.9±0.05
Piezoelectric modulus of elasticity, E_{pz} (*GPa*)	65±2
Piezoelectric density, ρ_{pz} (*kg/m*^{3})	7850
Electromechanical coupling coefficient, d_{31} (*pC/N*)	-190
Piezoelectric thickness, h_{pz} (*mm*)	0.8±0.05

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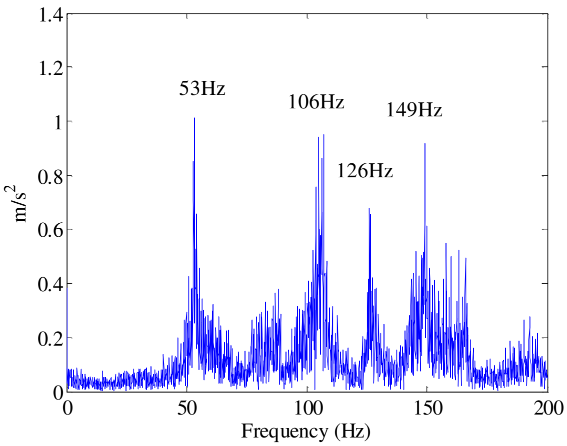

Fig. 3. Frequency response of the plate to a chirp excitation.

The highest peaks occur at 53 Hz, 106 Hz, 126 Hz and 149 Hz. Among these four modes, modes (1,1) and (1,3) that occur at 53 Hz and 106 Hz, respectively, have the highest peaks. The mode (1,2) at 80Hz does not provide an amplitude as high as modes (1,1) and (1,3). In the next section, the method of MPPF will be used to suppress the vibrations of one mode i.e., mode (1,1).

The control method is similar to PPF control method. The reason that piezoelectric patches are mounted at two locations (on the center and between center and the point of applying the disturbance) is simply because of interest in suppressing the vibrations at these two points, in particular at the center. The center is a point of interest since the highest amplitude of vibrations (vibrations at the first resonant frequency) occurs at the center of the clamped plate as shown in Fig. 1. The piezoelectric patch between the center and point of applying disturbance provides the ability to suppress the vibrations that come from the piezoelectric patch at the center and also the disturbance that come from the shaker. During the experimental procedure, the center piezoelectric patch will be adjusted first to control the vibration at the center, then the controller on the second piezoelectric patch is activated to suppress the remaining vibrations on the plate, especially at the point between the center and disturbance point.

In order to experimentally adjust the control gains, one should increase the gain of the first order compensator to provide a proper damping value and then increase the gain of the second order compensator to completely suppress the vibration. However, it should be noted that when more than one frequency is targeted to be suppressed, the compensator gains of higher frequency need to be adjusted first followed by adjusting lower frequencies to prevent the spillover effect.

IV. Results

In this section, the MPPF controller will be experimentally used to suppress the vibrations of the plate. In the first step, the first resonance frequency at 53 Hz will be excited, which has the highest amplitude of vibration, and MPPF controller will be applied to both piezoelectric patch to control the vibrations specifically at those two points but also all over the plate. In order to experimentally apply the MPPF method to the plate, the MPPF controller is designed, and the simulated using MATLAB® and Simulink® software. The controller then was converted to C® and uploaded onto a dSpace Autobox system. The sampling rate for the systems is set to be 0.0025 sec. The input voltage for both piezoelectric actuators and the shaker (to produce disturbance) and output signals from all the sensors are sent and received by the dSpace Autobox system.

To determine the control gains for one mode control, the shaker is set to produce a disturbance at 53 Hz, i.e., the first resonance frequency of the plate. Referring to equation (2), the **A** and **B** gain matrices for the case of one mode control are reduced to scalars, α_1 and β_1, respectively. The objective is to control the vibration at the center and then the rest of the plate, so the gains for the piezoelectric patch on the center are adjusted first, then the gains for the other piezoelectric patch. In the experimental one mode control, as the first step, the gain β_1 is adjusted to add damping into the system, then the gain of second order controller α_1 is adjusted. It should be noted that the gains are in the limit expressed in equation (9). If the conditions are similar to the case of $C_1 \approx \omega_1$, then, as explained at the end of Section 2 of this paper, the gains are limited to $(\alpha_1 + \beta_1) < 1$. It is recommended to start increasing the gain from zero to the point that the gain can produce the maximum vibration suppression, since after a certain point, increasing the gain does not provide better performance and even can increase the vibration or push the system near the instable region. Once the optimum gains for first piezoelectric patch are obtained, a similar procedure can be performed for the second piezoelectric patch. Figure 4 shows the control results at all 10 sensors on the plate, as shown and numbered in Fig.1. The shaker and the piezoelectric patches are deactivated at the beginning of the experiment. After 1.4 sec, the shaker is turned on and the vibrations are recorded by the sensors. When the response becomes stable, the controller of the first piezoelectric actuator (PZ 1) is activated after approximately 5 sec. It is seen that the vibrations in the center of the plate (Sensor No.1) have been very well suppressed by about 92% of the original amplitude. The vibrations have been also decreased at sensors 5, 6 and 9, but sensors 2, 4, 7, and 8 show increases of vibrations. Sensors 3 and 10 have not changed. By implementing the controller to PZ 2 at 7.6 sec, the vibrations have started to decrease in more sensors. When the controller is on for both piezoelectric patches the vibrations of all points have been decreased, except points 3 and 7. Sensor 3 is where the excitation has been applied and sensor 7 still has lower vibration amplitude than some other sensors. It should be noted that the main objective is vibration suppression of the center and then the other points and although the controller on PZ 2 can be actuated to provide better vibration suppression for point 2 and some other points, it causes marginal increase in the amplitude of vibrations of point 1 (center of the plate), which is not desired. For better observation of vibration suppression on the center of the plate, the frequency responses of this point before and after implementing control are shown in Fig. 5.

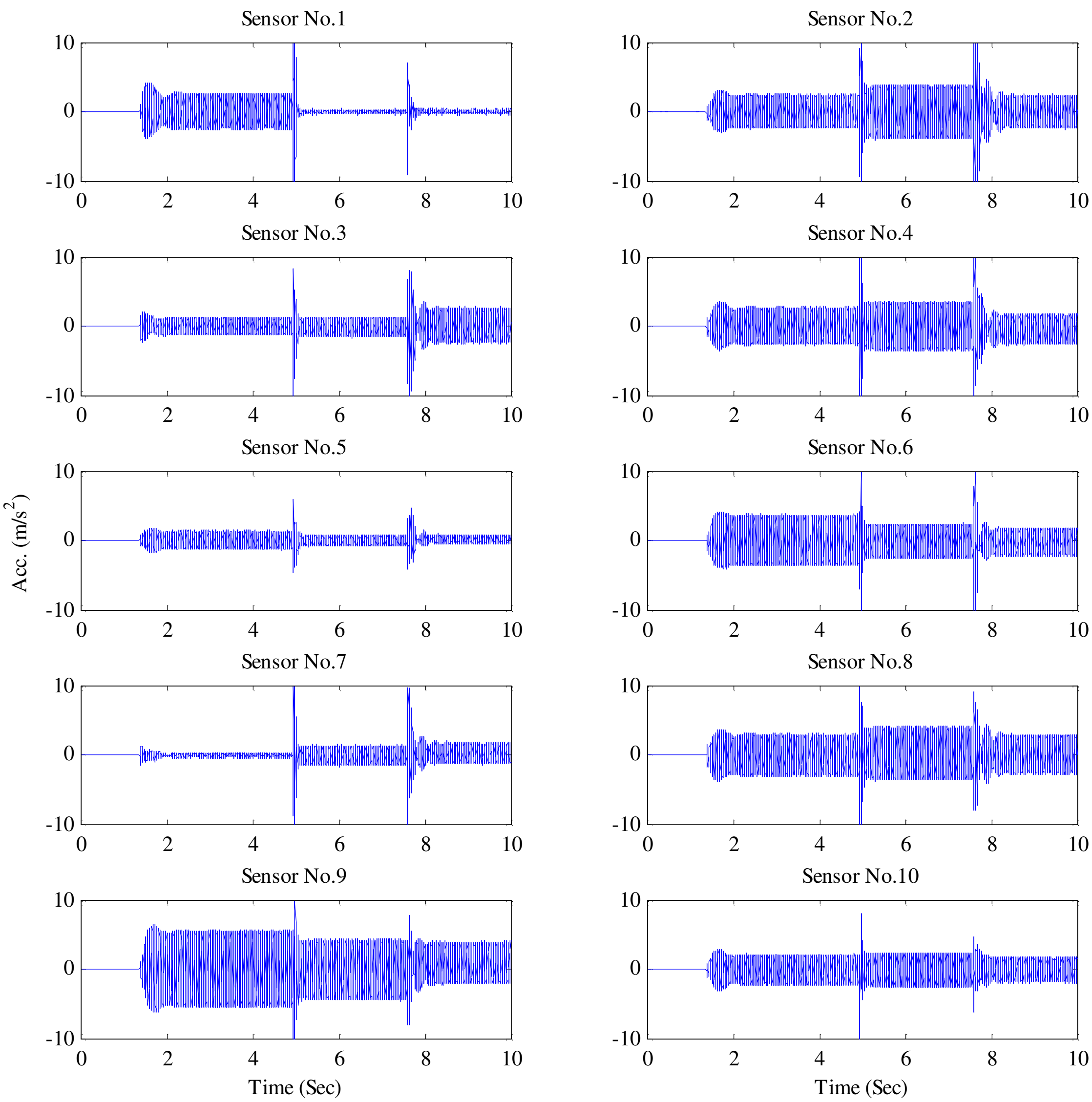

Fig. 4. Response of all 10 sensors on the plate without control, and after controlling with one and then two piezoelectric actuators.

Figure 5a shows the frequency response of the plate (response at the center, sensor No.1) when it is excited with the periodic disturbance at 53 Hz. Fig. 5b shows the frequency response of the same system when both controllers at PZ 1 and PZ 2 are activated. The response at 53 Hz is 19.96 dB for the system with no controller, while it is -37.34 dB when both controllers are on. It shows more than a 57 dB decrease in the amplitude of vibrations at the excitation frequency. At the frequency of 106 Hz, however, there is a large peak that has increased slightly after engaging the controller. Since the plate has been excited just with a 53 Hz periodic signal, and the 106 Hz peak is twice the first natural frequency, it may be concluded that there is nonlinear vibrations in the plate. The peak at 159 Hz (3×53 Hz) also confirms this assumption.

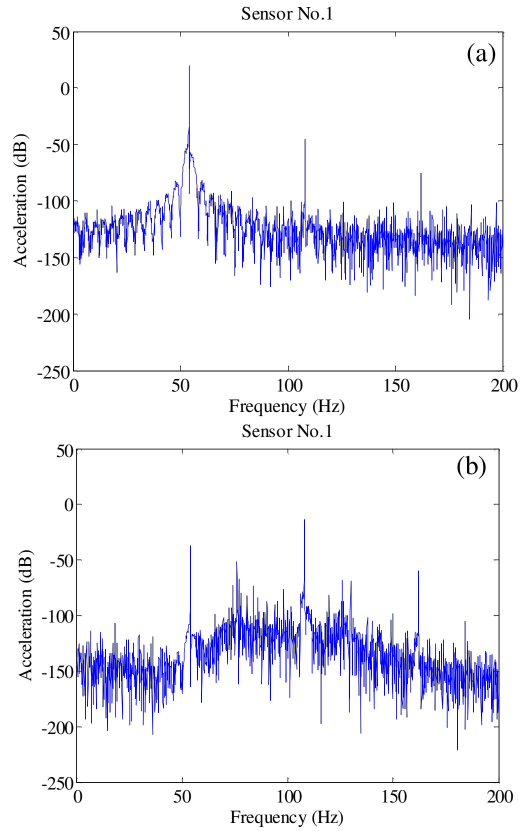

Fig. 5. Frequency response of Sensor No.1 to periodic disturbance with 53 Hz frequency; a) without control, b) controllers for both piezoelectric patches are on.

I. CONCLUSIONS

A new active vibration control method called Modified Positive Position Feedback (MPPF) was developed based on Positive Position Feedback and successfully verified experimentally. The controller consists of two parallel compensators: a first order compensator to provide damping and a second order compensator to suppress the vibration. The control effort is applied to the structure using two piezoelectric patches. The control gains were obtained using a stability analysis. A plate was used as the test platform for examining the controller. The dynamic model of the piezoelectrically actuated plate was constructed and the transfer function of the closed loop system was obtained. Using a chirp signal, the resonance frequencies of the plate were found and MPPF was used to suppress the vibrations of the fundamental resonant frequencies. The results show more than 90% suppression of vibration at the center of the plate for the first mode.

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