ABSTRACT
This article presents the results of two in-vivo studies providing measurements of human static ankle mechanical impedance. Accurate measurements of ankle impedance when muscles were voluntarily activated were obtained using a therapeutic robot, Anklebot, and an electromyographic recording system. Important features of ankle impedance, and their variation with muscle activity, are discussed, including magnitude, symmetry and directions of minimum and maximum impedance. Voluntary muscle activation has a significant impact on ankle impedance, increasing it by up to a factor of three in our experiments. Furthermore, significant asymmetries and deviations from a linear two-spring model are present in many subjects, indicating that ankle impedance has a complex and individually idiosyncratic structure. We propose the use of Fourier series as a general representation, providing both insight and a precise quantitative characterization of human static ankle impedance.

INTRODUCTION
Physical therapists employ motion therapy for a wide variety of physical ailments. The condition of the ankle (or other joints) is often characterized using a subjective scale of muscle tone called the Modified Ashworth Scale [1]. Here we describe an effort to analyze the features of a subject’s ankle impedance function (the relationship between the rotational motion of the foot relative to the shank and the torque exerted about the ankle joint) in a standardized and quantitative fashion using data collected from a therapeutic robot. With the advent of quantitative descriptions of ankle impedance, more understanding and insight into human physiology may emerge, describing not only the magnitude of ankle impedance, but also symmetry properties and directions of minimum and maximum impedance.

Sophisticated tools for measuring ankle impedance may improve clinical treatments and produce data useful for designing human-robot interaction. With objective and precise quantitative measurement, databases containing individual treatment histories as well as data on the general population can be established. The effects of various physical ailments and treatments can be compared among subjects, potentially improving clinical practice. Human-robot interaction may also benefit from a better quantification of human ankle impedance, enabling mobility assist devices and external robots to better predict and model the consequences of interacting physically with the human about the ankle joint.

Prior work has provided detailed quantitative assessment of multi-variable mechanical impedance about other joints in
the human body. The shoulder and elbow joints have been broadly studied, notably by Gomi and Osu [2]. That study showed that upper-limb impedance is configuration dependent, with stiffness ellipses varying depending on joint angle. However, it is unclear whether a stiffness ellipse provides a sufficiently general description of multi-variable static impedance, either in the upper limb or in other joints such as the wrist or ankle.

Significant efforts have been made to quantitatively measure ankle impedance, but the majority of studies have focused on the impedance along a single degree of freedom (DOF). Selles et al. and Yeh et al. measured passive ankle impedance in dorsiflexion and plantarflexion in stroke patients [1, 3]. Sinkjaer et al. evaluated ankle impedance in healthy subjects as a function of muscle activity in the dorsiflexion-plantarflexion directions [4]. Zinder et al. evaluated dynamic ankle impedance in the inversion-version directions [5]. Here we report measurements of static ankle impedance in combined degrees of freedom, and from the data suggest the use of Fourier series as an interpretive tool to assist the further quantification of ankle impedance. We also report the significant dependence of ankle static impedance on the voluntary modulation of muscle activation.

METHODS

We collected human ankle impedance data in two separate but related studies. The first study, A, (denoting active muscles) involved 8 unimpaired young subjects (4 male, 4 female). Ages ranged from 22-31 with a mean of 26.6 years. Weights ranged from 44.4kg to 84.8kg with a mean of 67.8kg. Height ranged from 1.52m to 1.91m with a mean of 1.72m. The second study, C, (denoting co-contraction of antagonist muscles) involved 8 unimpaired young subjects (5 male, 3 female). Ages ranged from 22-30 with a mean of 28 years. Weights ranged from 61kg to 100kg with an average of 74.5kg. Heights ranged from 1.651m to 1.86m with a mean of 1.75m. Subjects gave informed consent to participate in the study following procedures approved by MIT’s institutional review board.

The methodology differed slightly between study A and study C due to differences between the goals of the studies. In both studies, subjects were measured multiple times with a therapeutic robot. The first measurement was always accompanied with the instruction to fully relax and not interfere with the motion of the robot. This served as a baseline measurement. Surface electromyograms (EMG) were recorded to determine muscle activity levels. Electrodes were attached to the skin over the major superficial leg muscles: tibialis anterior, peroneus longus, soleus, and gastrocnemius. The EMG recordings were used to monitor and promote subjects’ adherence to experimental instructions.

The subsequent measurements varied according to experimental condition. In study A, subjects were asked to activate a single muscle at a constant EMG signal level. They were provided with visual feedback of the EMG signal in order to minimize variability. Activation of two muscles was studied: tibialis anterior (a dorsiflexor) and soleus (a plantarflexor). In study C, subjects were asked to activate several muscles at the same time, and to maintain the tibialis anterior and soleus at high activity levels (as indicated by EMG) while limiting their variability. The target EMG level for each subject was calibrated experimentally by finding the EMG amplitude corresponding to a sufficiently large target torque level for that muscle, at least 3.2 N-m in the direction of the muscle’s primary action.

Our primary tool, Anklebot, is a highly back-drivable therapeutic robot with three degrees of freedom, two of which are actuated, applying torques in the frontal and sagittal planes of the ankle but allowing free internal and external rotation. When worn, it allows researchers and clinicians to perturb the ankle joint without adding significant impedance to the joint. It is designed to provide up to 17 Nm of torque and to work with sitting, standing, or walking humans [6]. In our experiments, subjects wore Anklebot in a seated position. Anklebot was programmed to apply increasing and decreasing position perturbations to the foot. A proportional-plus-derivative (PD) position controller in joint space with low gains (Kp = 100 N-m/rad and Kd = 2 N-m-s/rad) was used to move the ankle along a commanded trajectory. The measured impedance of the ankle did not depend on the robot or controller gain, as the controller gain remained constant through the measurement. The position and torque histories imposed by this controller were recorded at 200Hz.

The measurement method used in this paper followed procedures previously reported by this group [7]. Terminated ramp perturbations were applied in 24 directions in the frontal and sagittal planes, with the direction of perturbation changing by 15 degrees on each ramp. To minimize the likelihood of evoking spindle-mediated stretch reflexes, the ramp velocity was set at 5 degrees per second, and ramps terminated at 20 degrees from the center position. A total of 48 movements, one increasing and one decreasing ramp per direction, were made per measurement. The foot was held briefly at the minimum or maximum of the ramp, for a period of 100ms. The foot was initially held by the robot with the sole at 90 degrees relative to the shank and with no inversion or eversion.

RESULTS

Raw data from the robot consisted of time-histories of commanded actuator forces and actuator lengths which were converted (as detailed in [6]) to torques about and angular displacements of the ankle. Torque and position data were analyzed piecewise for inbound and outbound displacement along each of the 24 directions. The static component of impedance for each direction was obtained by computing the slope of a linear least squares fit of the displacement and torque data in that direction (without constraining the line to intersect the origin and excluding the first 0.5 seconds of each ramp displacement to allow servo-induced transients to dissipate). By
sampling the impedance function along enough directions, the two-dimensional shape of ankle impedance emerged. Figures 1 and 2 show the measured values of ankle impedance (units of N-m per radian) under studies A and C respectively. The curves in the figures indicate the impedance of the ankle being perturbed from the neutral position in a particular orientation. In our numbering scheme, 0 degrees corresponds to a movement of pure eversion, while 90 degrees denotes pure dorsiflexion. There were eight measurements of eight unique subjects in study A in the passive condition and another eight measurements in the tibialis active condition. In the soleus active condition of study A there were seven measurements of seven subjects. In the passive and cocontraction conditions of study B, there were eight measurements from eight unique subjects.

**Figure 1. Study A: Mean Measured Passive and Active Impedance Values.** Thick Line Indicates Mean. Thin Line Indicates ± 1 Standard Error of the Mean. Solid Black Curve: Passive. Solid Light Curve: Soleus Active. Dashed Light Curve: Tibialis Anterior Active.

**Table 1. Average Measured Static Impedance from Study A**
(N-M/RAD, Mean ± Standard Error)

<table>
<thead>
<tr>
<th></th>
<th>Passive N=8</th>
<th>Tibialis Anterior Active N=8</th>
<th>Soleus Active N=7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eversion</td>
<td>7.60±0.89</td>
<td>11.24±1.92</td>
<td>11.50±1.94</td>
</tr>
<tr>
<td>Inversion</td>
<td>6.47±0.66</td>
<td>14.22±2.52</td>
<td>10.53±1.65</td>
</tr>
<tr>
<td>Dorsiflexion</td>
<td>15.77±2.41</td>
<td>33.27±6.89</td>
<td>38.64±8.40</td>
</tr>
<tr>
<td>Plantarflexion</td>
<td>16.76±2.73</td>
<td>28.29±4.96</td>
<td>29.86±3.95</td>
</tr>
</tbody>
</table>

**Figure 2. Study C: Mean Measured Passive and Co-contraction Impedance.** Thick Line Indicates Mean. Thin Line Indicates ± 1 Standard Error of the Mean. Black Curve: Passive. Light Curve: Co-contraction.

Certain directions are of particular interest: those corresponding to pure inversion, eversion, dorsiflexion, and plantarflexion have been previously reported in the literature. Table 1 shows the mean impedance values obtained from Study A in these cardinal directions of foot displacement. In the soleus active condition one subject exceeded the maximum torque capability of Anklebot; that data was excluded from the analysis. Table 2 shows the corresponding mean impedance values obtained from study C. Subjects were measured in only a single trial for these studies. Repeated measurements are planned for future studies. In study A, subjects increased their tibialis anterior EMG levels by a factor of 9.32 on average over the baseline (relaxed) condition, and they increased their soleus EMG levels by a factor of 5.58 over the baseline condition.

**Table 2. Average Measured Static Impedance from Study C**
(N-M/RAD, Mean ± Standard Error)

<table>
<thead>
<tr>
<th></th>
<th>Passive N=8</th>
<th>Cocontraction N=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eversion</td>
<td>8.80±0.98</td>
<td>17.58±2.40</td>
</tr>
<tr>
<td>Inversion</td>
<td>7.46±1.07</td>
<td>23.34±5.39</td>
</tr>
<tr>
<td>Dorsiflexion</td>
<td>17.91±2.49</td>
<td>69.22±7.80</td>
</tr>
<tr>
<td>Plantarflexion</td>
<td>16.66±3.08</td>
<td>52.23±10.2</td>
</tr>
</tbody>
</table>
DISCUSSION

The most obvious feature of Figure 1 and Figure 2 is that muscle activation significantly affects the magnitude of ankle impedance. In fact, impedance of any given subject’s ankle doubles or triples depending on the experimental condition. Under the conditions of our experiments, co-contraction of antagonist lower leg muscles increased ankle impedance the most. Table 3 presents the effect of muscle activation on the measured stiffness (the average factor of increase for all subjects in all directions of motion under that condition). The ability to alter ankle impedance voluntarily is important for humans to maintain stable postures and gaits, and our data show that sub-maximal activation of muscles gives humans access to a wide range of impedances.

### TABLE 3. MEAN RATIO OF ACTIVE IMPEDANCE VERSUS PASSIVE IMPEDANCE IN ALL DIRECTIONS

<table>
<thead>
<tr>
<th></th>
<th>Tibialis Active</th>
<th>Soleus Active</th>
<th>Cocontraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive</td>
<td>1.97</td>
<td>2.02</td>
<td>3.24</td>
</tr>
</tbody>
</table>

Another observation—equally true, though perhaps less obvious—is the inadequacy of stiffness ellipses to represent the static impedance of the ankle, even in the relaxed condition. A linear multivariable stiffness can be associated with an ellipsoid or an ellipse in 2D [8]. However, an ellipse is a strictly convex periodic function of angle with two orthogonal axes of symmetry. The ankle impedance shape is a periodic function, but it contains both concave and convex features and evidence of significant asymmetry. We propose the use of a finite Fourier series, Equation (1), to capture these features in a simple form. A Fourier series can exactly match any periodic function given sufficient terms. As shown in Figure 3, a third-order Fourier series captures at least 80% of the variance of the experimental data when its coefficients are determined using least squares fitting, and a fourth-order series offers little improvement (if any). Such a model (when constrained to have a period of $2\pi$) consists of seven parameters, a marked reduction from 24 data points.

$$F_n(\theta) = a_0 + a_1 \cos(\theta) + b_1 \sin(\theta) + \ldots + a_n \cos(n\theta) + b_n \sin(n\theta)$$  \hspace{1cm} (1)

By fitting all subjects’ data to third-order Fourier series approximations, it becomes possible to make comparisons and generalizations about the impedance function shape, independent of the magnitude of ankle impedance.

For example, the Fourier series representation enables reliable estimation of the directions in which ankle impedance is minimal and maximal (by searching for extreme values along the approximation.) We found that the local minima and maxima of ankle impedance were not exactly co-aligned with the cardinal directions, though the misalignment is modest (Table 4). Ankle impedance was lowest along the inversion-eversion axis, independent of muscle activation.

### FIGURE 3. Convergence of Fourier Series to Measured Data in Selected Subjects

### TABLE 4. DIRECTIONS OF LOCAL MINIMUM AND MAXIMUM ANKLE IMPEDANCE IN DEGREES

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>Max</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive (C study)</td>
<td>6.9</td>
<td>89.9</td>
<td>194.7</td>
<td>292.8</td>
</tr>
<tr>
<td>Cocontraction</td>
<td>0.6</td>
<td>95.4</td>
<td>194.1</td>
<td>273.4</td>
</tr>
<tr>
<td>Passive (A study)</td>
<td>9.6</td>
<td>93.0</td>
<td>187.1</td>
<td>284.1</td>
</tr>
<tr>
<td>TA Active</td>
<td>-3.4</td>
<td>98.5</td>
<td>163.1</td>
<td>283.7</td>
</tr>
<tr>
<td>S Active</td>
<td>5.4</td>
<td>92.7</td>
<td>175.0</td>
<td>251.0</td>
</tr>
</tbody>
</table>

The average magnitude of a particular subject’s impedance is captured in the zeroth-order term of the series. Normalizing with respect to this term gives the resulting impedance function a constant area. In polar coordinates, all even-subscript terms of the Fourier series will produce shapes that are symmetric about two perpendicular axes of symmetry. Odd-subscript terms, however, will produce shapes symmetric about only one axis. The magnitude of the odd-subscript terms can thus be used as a measure of the deviation of a subject’s ankle impedance function from bilateral symmetry.

Figure 4 shows the magnitude of the Fourier coefficients derived from both studies. The terms at each successive order are normalized by the zeroth-order term, enabling comparison between subjects with different body types, biomechanics and physiology. Note that most subjects exhibit significant first-order terms, indicating a significant deviation of their ankle impedance from symmetry about two axes.
As the odd Fourier coefficients represent asymmetry or skew of the impedance function, what might the even coefficients represent? Equations 2 and 3 show that the second-order term is consistent with a simple model of ankle impedance. Assume that ankle impedance is determined by two independent linear springs with stiffnesses, $K_{DP}$ and $K_{IE}$. $K_{DP}$ exerts a conservative force along the DP axis, proportional to $\sin(\theta)$. $K_{IE}$ exerts a conservative force along the IE axis, proportional to $\cos(\theta)$. Using our method of measuring and representing ankle impedance, the measured impedance is back-projected along the direction of displacement, thus yielding $\sin^2(\theta)$ along the DP axis and $\cos^2(\theta)$ along the IE axis (Equation 2). With a little algebra this can be shown to be equivalent to a constant (zeroth-order) term plus a second-order cosine term (Equation 3). Thus the zeroth-and second-order terms represent those components of ankle static mechanical impedance that can be attributed to a linear spring-like behavior.

$$F_{prof}(\theta) = K_{DP} \cdot \sin^2(\theta) + K_{IE} \cdot \cos^2(\theta)$$ (2)

$$F_{prof}(\theta) = \left( \frac{K_{DP}}{2} + \frac{K_{IE}}{2} \right) + \left( \frac{K_{IE} - K_{DP}}{2} \right) \cos(2\theta)$$ (3)

Much of the observed variance is consistent with a linear two-spring model. However, the presence of asymmetry in all muscle conditions indicates that some natural variance in human ankle impedance cannot be captured by a linear two-spring model.

The introduction of muscle activation and co-contraction does not significantly impact shape properties, but significantly changes the magnitude of ankle impedance. Therefore when humans modulate their muscle activity, they can affect the magnitude of impedance more than its shape (at least under the conditions of these experiments) and perturbations are likely to affect the foot in some directions (especially inversion-eversion) more than others.

Using a Fourier series as above provides a compact, intuitive, and convenient method for representing ankle impedance data, enabling further quantitative study of the biomechanics of the ankle.

ACKNOWLEDGMENTS

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REFERENCES


