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VIBRATION CONFINEMENT BY MINIMUM MODAL ENERGY EIGENSTRUCTURE ASSIGNMENT

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ABSTRACT

A novel Eigenstructure Assignment (ESA) method for vibration confinement of flexible structures has been developed. This method is an output feedback control and determines the closed-loop systems that their eigenvectors are orthogonalized to the open-loop eigenvectors. This method is a numerical method and used Singular Value Decomposition (SVD) to find the null space of the closed-loop eigenvectors. The matrix that spans the null space can be used to regenerate the open-loop system as well as the systems that have orthogonal eigenvectors to the regenerated open-loop system. As a result the isolation of vibration is independent of the type of the disturbance. Also in this method, the energy of the closed-loop system is minimized. As an important outcome, the proposed method needs neither to specify the closed-loop eigenvalues nor to define a desired set of eigenvectors.

INTRODUCTION

The idea of Eigenstructure Assignment (ESA) is given by Moore [1]. He characterized the class of all eigenvector sets related to a distinct set of closed-loop eigenvalues using state feedback [2]. Therefore, a control problem of eigenvalue placement for a MIMO system which had been introduced earlier by Wonham [3], had been redefined to both placement of eigenvalues in desired locations and choosing a set of the associated eigenvectors from a class of possible eigenvectors.

It was Cunningham who first used Singular Value Decomposition (SVD) to find the null space for the achievable eigenvector sub-space. In his output feedback control method, the basis vectors were optimally combined to minimize the error between achievable and desirable eigenvectors. This method was the first practical method of eigenstructure assignment in order to have a desirable transient response behavior [4]. Using SVD, a finite number of actuators are needed to shape the eigenvectors of the system [5].

Shelly et al studied the absolute displacement in first and second order systems, because the existing eigenstructure assignment would not have a control on them. They showed that it is not possible to tell if the absolute displacements in a system are increased, decreased or remained intact just by changing the system's eigenvectors [6]. Furthermore, they introduced a mode localization technique called eigenvector scaling while studying the time domain response of the system. This method changes specific elements of each eigenvector in order to uniformly decrease the relative displacement of the corresponding areas in the system [7]. They showed analytically that absolute displacements in isolated areas can be reduced by eigenvector shaping, regardless of the type of the disturbance. Some experimental results of eigenvector shaping have been reported in [2, 8, 9]. The eigenstructure shaping method is an active control method and is basically regenerating the behavior of the system when passive mode localization happens, by scaling and reforming part or all of the system mode shapes. Since all the shape modes are scaled in the same way, vibration confinement of the system is not affected by the type of disturbance. An application of this method is also reported in [10]. One of the drawbacks of the uniform scaling is that the number of needed actuator/sensor that has to be equivalent to the number of coupled modes of the system. It means that the action between neighboring systems has the key role in the number of actuators/sensors that are needed. SVD-eigenvector shaping has been introduced and used as a solution to the problem of limited actuators/sensors [11]. This method uses a Moore-Penrose generalized left inverse and produces the closest eigenvector in least square sense to the desired ones, since it gives the minimum Euclidean 2-norm error. This method allows use of fewer pairs of actuators and sensors than previous methods [11].

An active-passive hybrid vibration confinement system using piezoelectric network actuators has been proposed by Tang et al [12, 13]. Instead of the mechanical parts, the passive elements of the systems are the circuit inductors and resistors. This method finds optimal eigenvectors using a Rayleigh principle by minimizing the ratio of modal energy at the concerned area to the modal energy of the whole structure using an auxiliary eigenvalue problem. Therefore the need for pre-selecting the closed-loop eigenvectors is eliminated and the problem of closeness of the desired and achievable eigenvectors does not exist. A case study of this method has been presented in [14]

Pre-determination of the desired eigenvector components can cause unsatisfactory performance if a match between components of the desired and achievable eigenvectors happens in the unimportant degrees of freedom [12, 13]. Considering the problem of movement of neighborhood of the closed-loop eigenvalues, an eigenstructure method for constrained state or output feedback has been presented by Slater et al [15]. They showed when the eigenvectors are the only parameters that are required to be changed, the control efforts are not necessarily minimized if the closed-loop eigenvalues are being forced to be close to the open-loop eigenvalues. In fact, a large change in eigenvectors may need a large movement of the eigenvalues to minimize the feedback gains. They also showed that closed-loop eigenvalues and eigenvectors have to be consistent in order to avoid the large control efforts. Also, they proposed that since there is no method to have closed-loop eigenvectors and eigenvalues consistent, the minimum number of constraints should be imposed to the eigenvectors' elements in order to have a reasonable control effort.

The minimum modal energy eigenstructure assignment proposed in this literature addresses this problem. This novel method does not require specifying the locations for the closed-loop eigenvalues. Moreover, it does not need defining a desirable eigenvector. The closed-loop system has eigenvalues consistent with the closed-loop eigenvectors which are different from the open-loop eigenvalues.

This numerical eigenstructure assignment procedure uses an output feedback for controlling vibrations in flexible structures and is based on finding the closed-loop eigenstructures such that their eigenvectors are orthogonal to the open-loop eigenvectors. Almost all of the known eigenstructure assignment methods require a pre-determination of the eigenstructure or at least eigenvectors. A prior knowledge of the desired closed-loop system behavior in terms of the elements of its eigenvectors is not an easy task and from a practical point of view is very challenging. Predicting a desirable shape for the eigenvectors of a complicated system does not have a straightforward procedure. Especially, for the continuous system, increasing the model degrees of freedom makes the task of defining the desirable shape for eigenvectors even harder. Therefore the degrees of freedom of the models need to be kept as low as possible. The proposed method does not need a pre-determination of the closed-loop eigenstructure so a prior knowledge of the closed-loop system is not required. The flexibility of the available eigenstructure assignment methods which required designers to specify the desired closed-loop eigenvectors leads to an error caused by the difference between the desirable and admissible eigenvectors. The new method finds the admissible eigenvectors for the closed-loop system which are orthogonal to the open-loop eigenvectors. Therefore, there is virtually no limitation on the number of pairs of actuators and sensors as well as the model itself. Since the eigenvectors of the closed-loop system are admissible eigenvectors and also the closed-loop eigenvalues are consistent with them, the actuation forces are prevented to be large.

This new method is based on proving an interesting property in the null space generated by Singular Value Decomposition (SVD). The upper part of the matrix that spans the null space is known as the basis for the eigenvectors of the closed-loop system. This sub-matrix has the same row dimension as state matrix of the system does. Multiplying the conjugate transpose of this matrix to itself is basically the norm of the eigenvectors of the closed-loop system. This matrix product can be expressed as the modal energy of the system as well. This product has a unique property that its eigenvalues are zero and one. Zero eigenvalues for this matrix means zero modal energy. Using this property, the open-loop system can be regenerated by eigenvectors associated with the unity eigenvalue, and the eigenvectors associated with zero eigenvalues generate the closed-loop systems with eigenvectors orthogonal to the open-loop ones.

EIGENSTRUCTURE ASSIGNMENT PROBLEM DEFINITION

Consider the closed-loop equation of motion for a linear first order system

$$\{\dot{x}\} = [A + BKC]\{x\} + [E]\{f\} \tag{1}$$

where $[A]$ is the $2n \times 2n$ state matrix, $[B]$ is the $2n \times m$ input matrix, $[C]$ is the $m \times 2n$ output matrix, $[E]$ is the disturbance input matrix with $2n$ rows. $\{f\}$ is the disturbance vector. $[K]$ is $m \times m$ feedback gain matrix. $\{x\}$ is the $2n \times 1$ state vector and its time derivative is $\{\dot{x}\}$. The first n elements of the state vector are displacements and the last n elements are the velocities of the associated second-order system.

To eliminate the effect of the disturbance $\{f\}$ in the isolated area, a control gain matrix $[K]$ has to be found. The general eigenstructure assignment definition is to solve the following eigenvalue problem simultaneously for $[K]$ and ϕ_i

$$[A-\lambda_i I \quad | \, B]\begin{Bmatrix} \phi_i \\ KC\phi_i \end{Bmatrix} = 0 \quad i = 1, \ldots, 2n \tag{2}$$

where ϕ_i is the closed-loop eigenvectors of the system. λ_i is, generally, the closed-loop eigenvalues associated with ϕ_i, but in the proposed method is the open-loop eigenvalues and are called the operating eigenvalues. Also I is a $2n \times 2n$ identity matrix. Obviously the vector $\begin{Bmatrix} \phi_i \\ KC\phi_i \end{Bmatrix}$ is in the null space of the matrix $S_{\lambda i} = [A-\lambda_i I \quad | \, B]$. The null space of $S_{\lambda i}$ can be determined by applying SVD to $S_{\lambda i}$,

$$S_{\lambda i} = [A-\lambda_i I \quad | \, B] = [U_i][\Sigma_i \, | \, 0][V_i^*] \tag{3}$$

$[U_i]$ and $[V_i]$ are the left and right orthonormal matrices respectively. $[V_i^*]$ is the conjugate transpose of the complex matrix $[V_i]$. $[V_i]$ can be partitioned as

$$[V_i]_{(2n+m)\times(2n+m)} = \begin{bmatrix} [V_{11}^i]_{2n\times 2n} & [V_{12}^i]_{2n\times m} \\ [V_{21}^i]_{m\times 2n} & [V_{22}^i]_{m\times m} \end{bmatrix} \tag{4}$$

It is known that the second column block of the $[V_i]$ spans the null space of the $S_{\lambda i}$. Any linear combination of m columns of $[V_{12}^i]$ is an admissible eigenvector of the closed-loop system. Different methods have different approaches to find r^i. If r^i is determined, the desirable eigenvectors of the system is

$$\phi_i^a = [V_{12}^i]\{r^i\} \tag{5}$$

and the corresponding control gain matrix $[K]$ is determined by

$$KC\phi_i^a = [V_{22}^i]\{r^i\} \tag{6}$$

The proposed method uses the open-loop eigenvalues as operating eigenvalues and regenerates the open-loop system. At the same time it finds the space of all the eigenvectors that are orthogonal to the open-loop eigenvectors. Choosing the appropriate closed-loop eigenvectors leads to the appropriate gains needed for control system. Obviously, the closed-loop eigenvalues will be consistent with the closed-loop eigenvectors and do not have necessarily any relation to the open-loop eigenvalues. Therefore the proposed method does not need defining neither closed-loop eigenvalues nor eigenvectors.

MINIMUM MODAL ENERGY EIGENSTRUCTURE ASSIGNMENT BY MODE ORTHOGONALIZING

Using the achievable eigenvector definition, $\phi_i^a = [V_{12}^i]\{r^i\}$, The modal energy of the closed-loop system corresponding to the ith eigenvector of the closed-loop system can be written as

$$E_i = r^{i*} V_{12}^{i*} V_{12}^i r^i \tag{7}$$

The goal is to find a minimum level of modal energy corresponding to a concerned eigenvalue. Considering the null space of the eigenvectors associated with the operating eigenvalue λ_i

$$\mathbb{N}^i = \begin{Bmatrix} [V_{12}^i] \\ [V_{22}^i] \end{Bmatrix} \tag{8}$$

Norm of $\mathbb{N}^i$ is equal to one since $\mathbb{N}^i$ is the basis for the null space. $\left\|\mathbb{N}^i\right\|_2 = 1$. Therefore any row block of $\mathbb{N}$ has a norm of less than 1. So the magnitudes of their singular values belong to the interval $[0 \quad 1]$ [16].

$$\left\|[V_{12}^i]\right\| \le 1,\ [V_{12}^i]^* = \bar{U}^i \bar{S}^{i*} \bar{V}^{i*} \text{ and } \bar{S}^i \subseteq [0 \quad 1] \tag{9}$$

Also, since $[V_{12}^i]$ is a complex matrix, $[V_{12}^i]^*[V_{12}^i]$ is a Hermitian matrix. So,

$$[V_{12}^i]^*[V_{12}^i] = \bar{U}^i \bar{S}^{i*}\bar{S}^i\bar{U}^{i*} = \bar{U}^i \left|\bar{S}^i\right|^2 \bar{U}^{i*} = \bar{U}^i\bar{\Lambda}^i\bar{U}^{i*} \quad (10)$$

where $\bar{\Lambda}_i$ is the matrix of eigenvalues and $\bar{U}^i$ is the matrix of eigenvectors of $[V_{12}^i]^*[V_{12}^i]$. Equation (10) shows that the eigenvalues of $[V_{12}^i]^*[V_{12}^i]$ belong to the interval $[0 \;\; 1]$, since the absolute values of the singular values of $\bar{S}^i$ belong to this interval.

It can be shown that $[V_{22}^i]^*[V_{22}^i]$ has the same eigenvectors as $[V_{12}^i]^*[V_{12}^i]$ but its eigenvalues are different. More precisely, the summation of the eigenvalues of $[V_{12}^i]^*[V_{12}^i]$ and $[V_{22}^i]^*[V_{22}^i]$ associated with similar eigenvectors are unity.

Theorem:

Consider a $2n \times m$ non-square matrix $S_{\lambda i}$, $2n \geq m$ and. The null space of this matrix is spanned by the columns of a $(2n+m)\times m$ matrix $\mathbb{N}^i$. V_{12}^i is the upper $2n \times m$ sub-matrix of $\mathbb{N}^i$ and V_{22}^i is the lower $m \times m$ sub-matrix of $\mathbb{N}^i$. $V_{12}^{i*}V_{12}^i$ and $V_{22}^{i*}V_{22}^i$ have identical $m \times m$ eigenvector matrices and the summation of their ordered eigenvalue matrices is an $m \times m$ identity matrix.

Proof:

From equation (8) it can be written

$$\mathbb{N}_i^*\mathbb{N}_i = I \quad (11)$$

which can be expanded as

$$[V_{12}^i]^*_{2n\times 2n}[V_{12}^i]_{2n\times 2n} + [V_{22}^i]^*_{m\times m}[V_{22}^i]_{m\times m} = I \quad (12)$$

Equation (12) can be re-written using an eigenvalue decomposition of the left hand side Hermitian matrices.

$$\bar{U}^i(\bar{\Lambda}^i)\bar{U}^{i*} + \bar{U}_w^i(\bar{\Lambda}_w^i)\bar{U}_w^{i*} = I \quad (13)$$

where $\bar{\Lambda}_w^i$ and $\bar{U}_w^i$ are the eigenvalue and eigenvector matrices of $[V_{22}^i]^*[V_{22}^i]$. Pre-multiplying the aforementioned equation by $\bar{U}^{i*}$ and post multiplying by $\bar{U}^i$,

$$\bar{\Lambda}^i + \bar{U}^{i*}\bar{U}_w^i(\bar{\Lambda}_w^i)\bar{U}_w^{i*}\bar{U}^i = \bar{U}^{i*}I\bar{U}^i = I \quad (14)$$

which is equivalent to

$$(\bar{U}^{i*}\bar{U}_w^i)(\bar{\Lambda}_w^i)(\bar{U}^{i*}\bar{U}_w^i)^* = I - \bar{\Lambda}^i \quad (15)$$

The left hand side of the equation is basically an eigenvalue decomposition of the diagonal matrix $I - \bar{\Lambda}^i$. But the eigenvalue matrix of a diagonal matrix is the matrix itself. So

$$\bar{\Lambda}_w^i = I - \bar{\Lambda}^i \quad (16)$$

or

$$\bar{\Lambda}_w^i + \bar{\Lambda}^i = I \quad (17)$$

Also equation (15) holds if

$$\bar{U}^{i*}\bar{U}_w^i = I \quad (18)$$

which concludes

$$\bar{U}^i = \bar{U}_w^i \quad (19)$$

□

Using the theorem result and considering $\bar{\Lambda}_w^i$ the eigenvector matrix of $[V_{22}^i]^*[V_{22}^i]$, it can be written

$$[V_{22}^i]^*_{m\times m}[V_{22}^i]_{m\times m} = \bar{U}^i\Lambda_w^i\bar{U}^{i*} \quad (20)$$

where $\bar{\Lambda}_w^i$ satisfies equation (17).

Equation (10), the eigenvalue decomposition of $[V_{12}^i]^*[V_{12}^i]$, can be rewritten as

$$\bar{U}^{i*}[V_{12}^i]^*[V_{12}^i]\bar{U}^i = \bar{\Lambda}^i \quad (21)$$

Choosing the eigenvalue equal to unity from $\bar{\Lambda}^i$ and its corresponding eigenvector $\bar{U}_J^i$, it can be seen that

$$\bar{U}_J^{i*}[V_{12}^i]^*[V_{12}^i]\bar{U}_J^i = 1 \quad (22)$$

But $\bar{U}_J^i$ is associated with zero eigenvalue of $\bar{\Lambda}_w^i$.

$$\bar{U}_J^{i*}([V_{22}^i]^*[V_{22}^i])\bar{U}_J^i = 0 \quad (23)$$

Equation (23) holds if

$$[V_{22}^i]\bar{U}_J^i = 0 \quad (24)$$

which can be used to find the associated gain matrix.

$$KC\phi_i^a = [V_{22}^i]r^i = [V_{22}^i]\bar{U}_J^i = 0 \qquad (25)$$

Equation (25) means the gain matrix K becomes zero. Since there is no control gain, the open-loop system has been regenerated. Therefore $[V_{12}^i]\bar{U}_J^i$ is the eigenvector corresponding to the operating eigenvalue of the open-loop, since its norm is one and the gain associated with this eigenvector is zero.

The j th non-unity eigenvalue of $[V_{12}^i]^*[V_{12}^i]$, $\bar{\lambda}_j^i$, is several order of magnitudes smaller than unity. $\bar{U}_j^i$, the eigenvectors associated with $\bar{\lambda}_j^i$, is j th column of eigenvector matrix $\bar{U}^i$. If $\bar{U}_j^i$ is substituted as r^i the following equation can be concluded

$$r^{i*}[V_{12}^i]^*[V_{12}^i]r^i = \bar{\lambda}_j^i \cong 0 \qquad (26)$$

Comparing equation (26) to the modal energy equation (7), it can be concluded that for the coefficient vector $r^i = \bar{U}_j^i$, the modal energy of the i th mode is minimum $E_i \cong 0$.

Appending all the calculated closed-loop eigenvectors for all the modes that have been calculated, the following matrices can be written

$$V = \left[[V_{12}^1]r^1 \cdots [V_{12}^m]r^m \right] \qquad (27)$$

$$W = \left[[V_{22}^1]r^1 \cdots [V_{22}^m]r^m \right] \qquad (28)$$

Feedback gain matrix K is

$$K = W(CV)^{-1} \qquad (29)$$

The state matrix for the closed-loop system is defined as

$$A_c = A + BKC \qquad (30)$$

Since the actuators and sensors are collocated, the matrix product BKC has zero elements on its diagonal and its trace is zero. As a result the summations as well as the average of the eigenvalues for the open-loop and closed-loop systems are equal.

Since the system has m inputs, there are m different modes that their null space has to be found. Also $[V_{12}^i]^*[V_{12}^i]$ $i = 1 \cdots m$ has m different eigenvectors that can be used as the coefficient vector r^i . Excluding the case that the open-loop system has been regenerated, there are $m^m - 1$ options for minimum modal energy ESA. The best solution is the one which has the smallest phase plane of isolated states.

CASE STUDY: A SYSTEM WITH 3 COLLOCATED ACTUATORS AND SENSORS

A simple lumped longitudinal vibration system has been considered as Figure 1 and the minimum energy eigenstructure assignment using mode orthogonalizing method has been applied in order to isolate the left side of the system from the vibration.

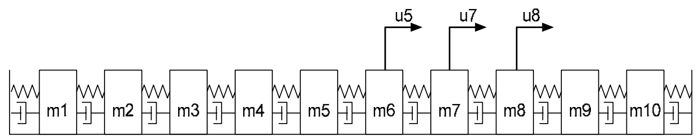

Figure 1. The system of 10 masses with interconnecting springs and dampers.

The system consists of 10 masses which are interconnected by springs and dampers as indicated in Figure 1. The goal is to isolate m_{1-5} while a chirp input is applied to m_{9-10} .Location of the actuators separates the isolated area from the disturbed area.

It is assumed that all the masses are equal to 50 kg . Also, all the spring coefficients are identical and are equal to 1000 N/m . Damping coefficients are assumed to be 10 $N.s/m$.

The core of the minimum modal energy algorithm is the control of the system in such a way that the closed-loop eigenvectors become orthogonal to the open-loop ones. This procedure is explained using a few examples.
Considering this system to have three pairs of collocated actuators and sensors which are located on m_6 , m_7 and m_8 . All the element of the B and C matrices in equation (1) are zero except

$$B(16,1) = B(17,2) = B(18,3) = -1/50$$
$$C(1,6) = C(2,7) = C(3,8) = 1$$

Operating eigenvalues, based on the step 1 of the procedure are λ_1, λ_3 and λ_5 . Therefore the problem is finding the appropriate r^1 , r^2 and r^3 for $V_{12}^1 r^1$, $V_{12}^2 r^2$ and $V_{12}^3 r^3$, respectively. Two different cases with different values for r^i associated with each operating eigenvalues are considered.

Case 1:
$r^1 = \bar{U}_3^1$, $r^2 = \bar{U}_3^2$ and $r^3 = \bar{U}_3^3$, which gives the zero gain matrix. The closed-loop system is a regeneration of the open-loop system.

Case 2:
$r^1 = \bar{U}_2^1$, $r^2 = \bar{U}_1^2$ and $r^3 = \bar{U}_1^3$, which generates the best vibration confinement.

For case 1, with the first open-loop eigenvalue, λ_1, the following equations can be written

$$V_{12}^{1*}V_{12}^1 = \begin{bmatrix} 0.4137 & -0.3793 & 0.3140 \\ -0.3793 & 0.3479 & -0.2881 \\ 0.3140 & -0.2881 & 0.2387 \end{bmatrix}$$

$$= \bar{U}^1\bar{\Lambda}^1\bar{U}^{1*} = \begin{bmatrix} 0.3723 & -0.6692 & 0.6431 \\ 0.7982 & -0.1228 & -0.5898 \\ 0.4737 & 0.7329 & 0.4884 \end{bmatrix}\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & 0.0002 & \vdots \\ 0 & \cdots & 1 \end{bmatrix}\begin{bmatrix} 0.3723 & 0.7982 & 0.4737 \\ -0.6692 & -0.1228 & 0.7329 \\ 0.6431 & -0.5898 & 0.4884 \end{bmatrix}$$

Also

$$V_{22}^{1*}V_{22}^1 = \begin{bmatrix} 0.5863 & 0.3793 & -0.3140 \\ 0.3793 & 0.6521 & 0.2881 \\ -0.3140 & 0.2881 & 0.7613 \end{bmatrix}$$

$$= \bar{U}^1\bar{\Lambda}_w^1\bar{U}^{1*} = \begin{bmatrix} 0.3723 & -0.6692 & 0.6431 \\ 0.7982 & -0.1228 & -0.5898 \\ 0.4737 & 0.7329 & 0.4884 \end{bmatrix}\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & 0.9998 & \vdots \\ 0 & \cdots & 0 \end{bmatrix}\begin{bmatrix} 0.3723 & 0.7982 & 0.4737 \\ -0.6692 & -0.1228 & 0.7329 \\ 0.6431 & -0.5898 & 0.4884 \end{bmatrix}$$

It is seen that the eigenvectors of the $V_{12}^{1*}V_{12}^1$ and $V_{22}^{1*}V_{22}^1$ are equal and also

$$\bar{\Lambda}_w^1 + \bar{\Lambda}^1 = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & 0.9998 & \vdots \\ 0 & \cdots & 0 \end{bmatrix} + \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & 0.0002 & \vdots \\ 0 & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & 1 & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$$

If the first column eigenvector matrix is chosen, the first column of W becomes zero.

$$V_{22}^1 r^1 = V_{22}^1\bar{U}_3^1 = \begin{bmatrix} 0.5872 & 0.3793 & -0.3151 \\ 0.3785 & 0.6521 & 0.2890 \\ -0.3135 & 0.2881 & 0.7606 \end{bmatrix}\begin{bmatrix} 0.6431 \\ -0.5898 \\ 0.4884 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Similar results can be found for λ_3 and λ_5. W becomes a zero matrix which leads to a zero gain matrix and, the open-loop and closed-loop systems become identical

For case 2, which is the most efficient closed-loop system, the following equations can be written

$$V = \left[[V_{12}^1]\bar{U}_2^1 \quad [V_{12}^2]\bar{U}_1^2 \quad [V_{12}^3]\bar{U}_1^3 \right]$$
$$W = \left[[V_{22}^1]\bar{U}_2^1 \quad [V_{22}^2]\bar{U}_1^2 \quad [V_{22}^3]\bar{U}_1^3 \right]$$

and real gain matrix is

$$K = W(CV)^{-1} = 1.0\times10^3\begin{bmatrix} -1.3205 & -0.8670 & -0.1337 \\ 1.5734 & -1.5268 & 0.1758 \\ 0.7264 & -1.0190 & -0.8965 \end{bmatrix}$$

Figure 2 shows the displacements of the masses due to a chirp input which rises from 0 to 100 N in 1 second and then becomes zero. Inputs are applied to m_9 and m_{10} while the actuators and sensors are on $m_{6,7,8}$. A great isolation can be seen on m_{1-5}. Because of vibration confinement, the energy that entered the system does not propagate beyond the actuators. The displacement of m_{9-10} that belong to the confined are increased. $m_{7,8}$ have almost similar amplitude of vibrations as open-loop system, but m_6 which carries the inner actuator has shown an isolated behavior.

The time histories of the displacement of m_1 are presented in Figure 3 for different cases. A unit impulse input is applied on m_{10}. It is seen that the closed-loop response of y_1 is the same as the open-loop response in case 1, while vibration is isolated in case 2.

Eigenvalues of the closed-loop system as well as the open-loop ones are presented in Figure 4. For both case 1 and case 2, the averages of the poles are -0.3. The reason is that the state matrices A and $A+BKC$ have the same diagonal elements. Because of the collocation of the actuators and sensors, the product BKC has zero elements on its diagonal.

Figure 5 shows the frequency responses of the masses due to the input at m_9. Frequency responses and the frequencies are in logarithmic scales. It can be seen that the magnitudes of the responses of $m_{1\text{-}8}$ for the closed-loop system are significantly smaller than those of the open-loop system at a range of frequency which consists of all the natural frequencies of the open-loop and closed-loop systems. The frequency responses, however, for the closed-loop system are larger than the open-loop ones at the frequencies larger than the largest natural frequency of the system. For example, at a frequency of about 20 rad/s, frequency response y_1 of the closed-loop system becomes larger than the open-loop response, while the maximum natural frequencies of the open-loop and closed-loop systems are 8.85 rad/s and 9.23 rad/s respectively. At this frequency, the amplitude of the response is too small and negligible.

CONCLUSION

The eigenstructure assignment method that is introduced here minimizes the modal energy of the closed-loop system, by orthogonalizing the closed-loop system eigenvectors to the open-loop system ones. This method is an output feedback control that can be applied to linear time invariant systems. The actuators and the sensors need to be collocated. This method uses the singular value decomposition to find the null space of the eigenvectors of the closed-loop system.

Conventional eigenstructure assignment methods need to define the desirable eigenvectors. Usually the desirable eigenvectors do not lie within the admissible sub-space of the eigenvectors, therefore some errors are unavoidable. Minimum modal energy ESA does not require defining the desirable eigenvectors. This method finds closed-loop systems that their eigenvectors are within admissible eigenvector subspace which

are orthogonal to the open loop eigenvectors. As a result the isolation is not depended on the type of the disturbance. Also, this new method does not specify a location for the closed-loop eigenvalues; therefore the extra constraints are not imposed to the actuators and the actuation forces can be reduced. In summary this method does not need any prediction about the closed-loop behavior and the results of the identification of the open-loop systems are the only data that are used in the design of the control algorithm.

It has been shown that the vibrational energy is being confined to the designated area, and the desired area has been successfully isolated while the amplitude of vibration in the confined area has been increased. Since the actuators and sensors are collocated, this method keeps the average of the closed-loop eigenvalues the same as open-loop ones.

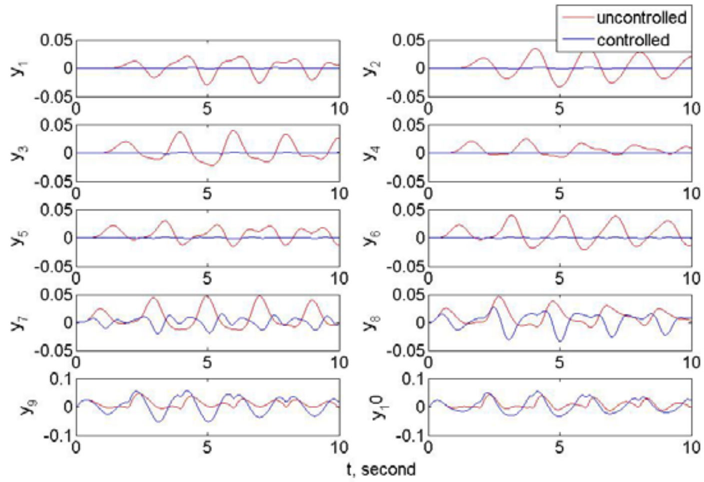

Figure 2. Displacement of the masses for open-loop and closed-loop systems due to a chirp input to $m_{9\text{-}10}$

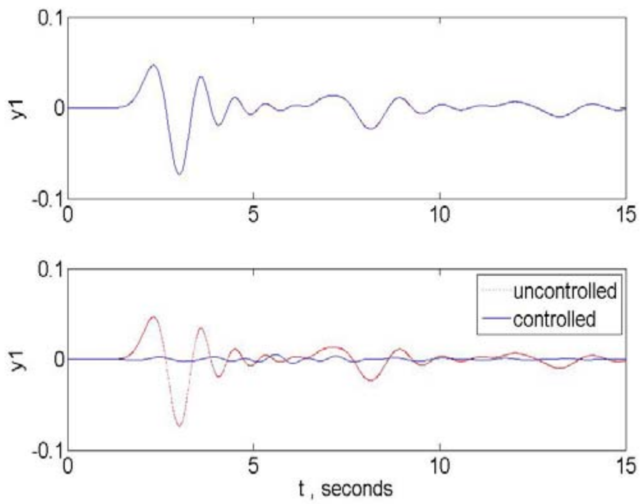

Figure 3. a- displacement of m_1 for regenerated system, case1, b- comparison of displacements of m_1, case 2

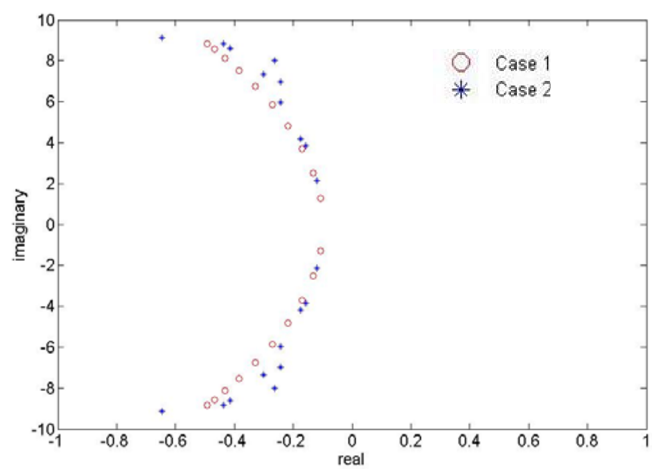

Figure 4. Eigenvalues of the closed-loop and the open-loop systems

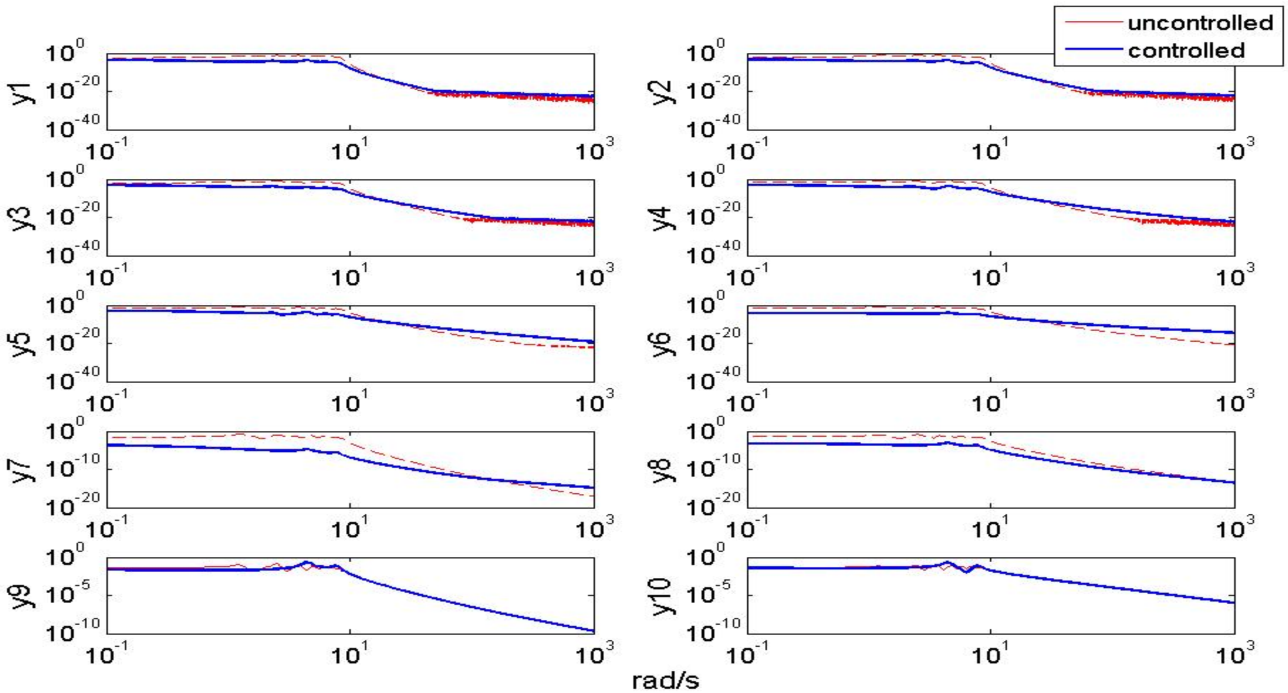

Figure 5. Frequency response of the masses for open-loop and closed-loop systems due to an input to m_9

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