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EFFECT OF THE ACTUATORS' LOCATION ON VIBRATION SUPPRESSION USING MINIMUM MODAL ENERGY EIGENSTRUCTURE ASSIGNMENT

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ABSTRACT

A new Eigenstructure Assignment (ESA) method for vibration confinement of flexible structures has been developed. This method is based on finding an output feedback control gain matrix in such a way that the closed-loop eigenvectors are orthogonal to the open-loop ones. Singular Value Decomposition (SVD) is used for finding the matrix that spans the null space of the closed-loop eigenvectors. It is shown that this matrix has a unique property that can be used to regenerate the open-loop system. This method finds a coefficient vector which leads to a zero gain matrix while several coefficient vectors can be found simultaneously which are orthogonal to the open-loop coefficient vector. As a result, the closed-loop eigenvectors are orthogonal to the open-loop ones. It is shown that the modal energy of the closed loop system is reduced. Moreover, the proposed method needs neither to specify the closed-loop eigenvalues nor to define a desired set of eigenvectors. Also it is shown that if the maximum force of the actuators and the consumed energy of the actuators need to be low, actuators have to be relatively close to input. If the amplitude of vibration in isolated area has to be minimized as much as possible, the actuators need to be relatively closer to isolated area. Also the algorithm of the minimum eigenstructure assignment method has been modified to eliminate the effect of the actuators that are located on the nodes of different vibrational modes.

INTRODUCTION

Eigenstructure Assignment (ESA) was first introduced by Moore [1]. Moore determined the class of all eigenvectors associated with the closed-loop eigenvalues using a state feedback control. In fact, the control of a MIMO system considers both eigenvalue placement and defining the associated eigenvectors form a class of possible closed-loop eigenvectors [2]. To define the null space of the achievable eigenvectors, Cunningham used Singular Value Decomposition (SVD) [3]. It had been shown that the number of actuators is finite, for suppression of structural vibrations, if SVD is used [4].

Eigenvector scaling, a mode localization technique, has been introduced by Shelly et al. Eigenvector scaling changes the elements of eigenvectors in such a way that relative displacement within the area corresponding to those elements decrease [5]. They showed that eigenvector shaping reduces vibration in the isolated area regardless of the disturbance type. Some experimental works have been done using the eigenvector shaping and reported in [2, 6-8]. Eigenvector shaping using SVD has been introduced and used as a solution to the problem of limited pairs of actuators and sensors in [9]. A Moore-Penrose generalized left inverse is used to produce the closest eigenvector in least square sense to the desired ones, since it gives the minimum Euclidean 2-norm error [9, 10].

Tang et al proposed a method that uses the piezoelectric network actuators in an active-passive hybrid vibration confinement system [11, 12]. The energy is confined to the circuit inductors and resistors as passive elements of the systems rather than bulky mechanical parts. Using the Rayleigh principle optimal eigenvectors can be found. Their method minimizes the ratio of the modal energy at the concerned area to the modal energy of the whole structure using an auxiliary eigenvalue problem [13].

Slater et al [14] showed when the eigenvectors are required to be changed and the closed-loop eigenvalues are being forced to be close to the open-loop eigenvalues, the control efforts are not necessarily minimized. In fact, a considerable change in the eigenvectors may require a large shift in the locations of the eigenvalues in order to minimize the feedback gains. Obviously the closed-loop eigenvalues and eigenvectors must be consistent in order to avoid the large control efforts. Also, they proposed that the minimum number of constraints should be imposed to the eigenvectors' elements in order to have a reasonable control effort. The minimum modal energy eigenstructure assignment proposed in this literature addresses this problem. This novel method does not require specifying the locations for the closed-loop eigenvalues. Moreover, it does not need defining a desirable eigenvector. After defining the closed-loop system eigenvectors, the eigenvalues consistent with the closed-loop eigenvectors will be found which are different from the open-loop eigenvalues.

The numerical eigenstructure assignment procedure proposed in this paper uses output feedback for controlling vibrations in flexible structures. This minimum modal energy eigenstructure assignment finds the closed-loop eigenstructures such that their eigenvectors are orthogonal to the open-loop ones. Conventional eigenstructure assignment methods require defining a desirable eigenvector. From a practical point of view pre-determining the desired eigenvectors is not simple since there is not a general procedure for this purpose. This task becomes even more challenging when the model degrees of freedom increase. In this new method, the practical limitation on the degrees of freedom of the models is eliminated since there is no need for defining the desired eigenvectors. This method finds the admissible eigenvectors for the closed-loop system that are orthogonal to the open-loop eigenvectors. As a result there is virtually no limitation on the number of actuators. Since the eigenvectors of the closed-loop systems are admissible eigenvectors and the closed-loop eigenvalues are consistent with them, there is no extra constraint for defining the eigenstructure; therefore, the excessive actuation forces are prevented [14].

In this paper, six different cases that have different formation of actuators and sensors have been studied. Also, the algorithm for minimum modal energy eigenstructure assignment method has been extended in such a way that the number of possible closed-loop systems is increased.

EIGENSTRUCTURE ASSIGNMENT PROBLEM DEFINITION

Consider the closed-loop equation of motion for a linear first order system

$$\{\dot{x}\} = [A + BKC]\{x\} + [E]\{f\} \qquad (1)$$

where $[A]$ is the $2n \times 2n$ state matrix, $[B]$ is the $2n \times m$ input matrix, $[C]$ is the $m \times 2n$ output matrix, $[E]$ is the disturbance input matrix with $2n$ rows. $\{f\}$ is the disturbance vector. $[K]$ is $m \times m$ feedback gain matrix. $\{x\}$ is the $2n \times 1$ state vector and its time derivative is $\{\dot{x}\}$. The first n elements of the state vector are displacements and the last n elements are the velocities of the associated second-order system.

To eliminate the effect of the disturbance $\{f\}$ in the isolated area, a control gain matrix $[K]$ has to be found. The general eigenstructure assignment definition is to solve the following eigenvalue problem simultaneously for $[K]$ and ϕ_i

$$[A - \lambda_i I \quad | B]\begin{Bmatrix} \phi_i \\ KC\phi_i \end{Bmatrix} = 0 \quad i = 1,\ldots,2n \qquad (2)$$

where ϕ_i is the closed-loop eigenvectors of the system. λ_i is , generally, the closed-loop eigenvalues associated with ϕ_i, but in the proposed method is the open-loop eigenvalues and is called operating eigenvalue. Also I is a $2n \times 2n$ identity matrix. Obviously the vector $\begin{Bmatrix} \phi_i \\ KC\phi_i \end{Bmatrix}$ is in the null space of the matrix $S_{\lambda i} = [A - \lambda_i I \quad | B]$. The null space of $S_{\lambda i}$ can be determined by applying SVD to $S_{\lambda i}$,

$$S_{\lambda i} = [A - \lambda_i I \quad | B] = [U_i][\Sigma_i \,|\, 0][V_i^*] \qquad (3)$$

$[U_i]$ and $[V_i]$ are the left and right orthonormal matrices respectively. $[V_i^*]$ is the conjugate transpose of the complex matrix $[V_i]$. $[V_i]$ can be partitioned as

$$[V_i]_{(2n+m)\times(2n+m)} = \begin{bmatrix} [V_{11}^i]_{2n\times 2n} & [V_{12}^i]_{2n\times m} \\ [V_{21}^i]_{m\times 2n} & [V_{22}^i]_{m\times m} \end{bmatrix} \qquad (4)$$

It is known that the second column block of the $[V_i]$ spans the null space of the $S_{\lambda i}$. Any linear combination of m columns of $[V_{12}^i]$ is an admissible eigenvector of the closed-loop system. Different methods have different approaches to find r^i. If r^i is determined, the desirable eigenvectors of the system is

$$\phi_i^a = [V_{12}^i]\left\{r^i\right\} \qquad (5)$$

and the corresponding control gain matrix $[K]$ is determined by

$$KC\phi_i^a = [V_{22}^i]\left\{r^i\right\} \qquad (6)$$

MINIMUM MODAL ENERGY EIGENSTRUCTURE ASSIGNMENT BY MODE ORTHOGONALIZING

Using the achievable eigenvector definition, $\phi_i^a = [V_{12}^i]\left\{r^i\right\}$, The modal energy of the closed-loop system corresponding to the i th eigenvector of the closed-loop system can be written as

$$E_i = r^{i*}V_{12}^{i*}V_{12}^i r^i \qquad (7)$$

The goal is to find a minimum level of modal energy corresponding to a concerned eigenvalue. Considering the null space of the eigenvectors associated with the operating eigenvalue λ_i

$$\mathbb{N}^i = \begin{Bmatrix} [V_{12}^i] \\ [V_{22}^i] \end{Bmatrix} \qquad (8)$$

Norm of $\mathbb{N}^i$ is equal to one since $\mathbb{N}^i$ is the basis for the null space. $\left\|\mathbb{N}^i\right\|_2 = 1$. Therefore any row block of $\mathbb{N}$ has a norm of less than 1. So the magnitudes of their singular values belong to the interval $[0 \quad 1]$ [15].

$$\left\|[V_{12}^i]\right\| \le 1,\ [V_{12}^i]^* = \bar{U}^i \bar{S}^{i*} \bar{V}^{i*} \text{ and } \bar{S}^i \subseteq [0 \quad 1] \qquad (9)$$

Also, since $[V_{12}^i]$ is a complex matrix, $[V_{12}^i]^*[V_{12}^i]$ is a Hermitian matrix. So,

$$[V_{12}^i]^*[V_{12}^i] = \bar{U}^i \bar{S}^{i*} \bar{S}^i \bar{U}^{i*} = \bar{U}^i \left|\bar{S}^i\right|^2 \bar{U}^{i*} = \bar{U}^i \bar{\Lambda}^i \bar{U}^{i*} \qquad (10)$$

where $\bar{\Lambda}_i$ is the matrix of eigenvalues and $\bar{U}^i$ is the matrix of eigenvectors of $[V_{12}^i]^*[V_{12}^i]$. Equation (10) shows that the eigenvalues of $[V_{12}^i]^*[V_{12}^i]$ belong to the interval $[0 \quad 1]$, since the absolute values of the singular values of $\bar{S}^i$ belong to this interval.

It can be shown that $[V_{22}^i]^*[V_{22}^i]$ has the same eigenvectors as $[V_{12}^i]^*[V_{12}^i]$ but its eigenvalues are different. More precisely, the summation of the eigenvalues of $[V_{12}^i]^*[V_{12}^i]$ and $[V_{22}^i]^*[V_{22}^i]$ associated with similar eigenvectors are unity. Using the null space property

$$\mathbb{N}_i^* \mathbb{N}_i = I \qquad (11)$$

It can be re-written as

$$[V_{12}^i]_{2n\times 2n}^*[V_{12}^i]_{2n\times 2n} + [V_{22}^i]_{m\times m}^*[V_{22}^i]_{m\times m} = I \qquad (12)$$

Since each term is a Hermitian matrix, they can be written in terms of their eigenvalue decompositions

$$\bar{U}^i(\bar{\Lambda}^i)\bar{U}^{i*} + \bar{U}_w^i(\bar{\Lambda}_w^i)\bar{U}_w^{i*} = I \qquad (13)$$

Pre-multiplying equation (13) by $\bar{U}^{i*}$ and post multiplying by $\bar{U}^i$,

$$\bar{\Lambda}^i + \bar{U}^{i*}\bar{U}_w^i(\bar{\Lambda}_w^i)\bar{U}_w^{i*}\bar{U}^i = \bar{U}^{i*} I \bar{U}^i = I \qquad (14)$$

or

$$(\bar{U}^{i*}\bar{U}_w^i)(\bar{\Lambda}_w^i)(\bar{U}^{i*}\bar{U}_w^i)^* = I - \bar{\Lambda}^i \qquad (15)$$

The left hand side of the equation is basically an eigenvalue decomposition of the diagonal matrix $I - \bar{\Lambda}^i$. But the eigenvalue matrix of a diagonal matrix is the matrix itself. So

$$\bar{\Lambda}_w^i + \bar{\Lambda}^i = I \qquad (16)$$

$$\bar{U}^i = \bar{U}_w^i \qquad (17)$$

The eigenvalue decomposition of $[V_{22}^i]^*[V_{22}^i]$ is

$$[V_{22}^i]_{m\times m}^*[V_{22}^i]_{m\times m} = \bar{U}^i \Lambda_w^i \bar{U}^{i*} \qquad (18)$$

where $\bar{\Lambda}_w^i$ satisfies equation (16).

Using the eigenvalue decomposition of $[V_{12}^i]^*[V_{12}^i]$, it can be rewritten

$$\bar{U}^{i*}[V_{12}^i]^*[V_{12}^i]\bar{U}^i = \bar{\Lambda}^i \qquad (19)$$

Choosing the eigenvalue equal to unity from $\bar{\Lambda}^i$ and its corresponding eigenvector $\bar{U}_J^i$, it can be seen that

$$\bar{U}_J^{i*}[V_{12}^i]^*[V_{12}^i]\bar{U}_J^i = 1 \quad (20)$$

But $\bar{U}_J^i$ is associated with zero eigenvalue of $\bar{\Lambda}_w^i$.

$$\bar{U}_J^{i*}([V_{22}^i]^*[V_{22}^i])\bar{U}_J^i = 0 \quad (21)$$

which leads to

$$[V_{22}^i]\bar{U}_J^i = 0 \quad (22)$$

which can be used to find the associated gain matrix.

$$KC\phi_i^a = [V_{22}^i]r^i = [V_{22}^i]\bar{U}_J^i = 0 \quad (23)$$

Equation (23) states the gain matrix K becomes zero. Since there is no control gain, the open-loop system has been regenerated. Therefore, $[V_{12}^i]\bar{U}_J^i$ is the eigenvector corresponding to the operating eigenvalue of the open-loop, since its norm is one and the gain associated to this eigenvector is zero.

The j th non-unity eigenvalue of $[V_{12}^i]^*[V_{12}^i]$, $\bar{\lambda}_j^i$, is several order of magnitudes smaller than unity. $\bar{U}_j^i$, the eigenvector associated with. $\bar{\lambda}_j^i$, is j th column of eigenvector matrix $\bar{U}^i$. If $\bar{U}_j^i$ is substituted as r^i the following equation can be concluded

$$r^{i*}[V_{12}^i]^*[V_{12}^i]r^i = \bar{\lambda}_j^i \cong 0 \quad (24)$$

Comparing equation (24) to the modal energy equation (7), it can be concluded that for the coefficient vector $r^i = \bar{U}_j^i$, the modal energy of the i th mode is minimum $E_i = 0$.

Appending all the calculated closed-loop eigenvectors for all the modes that have been calculated, the following matrices can be written

$$V = \left[[V_{12}^1]r^1 \cdots [V_{12}^m]r^m\right] \quad (25)$$

$$W = \left[[V_{22}^1]r^1 \cdots [V_{22}^m]r^m\right] \quad (26)$$

Feedback gain matrix K is

$$K = W(CV)^{-1} \quad (27)$$

The state matrix for the closed-loop system is defined as

$$A_c = A + BKC \quad (28)$$

Since the actuators and sensors are collocated, the matrix product BKC has zero elements on its diagonal and its trace is zero. As a result the summations as well as the average of the eigenvalues for the open-loop and closed-loop systems are equal.

The system has m inputs, so; there are m different modes that their null space has to be found. Also $[V_{12}^i]^*[V_{12}^i]$ $i = 1 \cdots m$ has m different eigenvectors that can be used as the coefficient vector r^i. Excluding the case that the open-loop system has been regenerated, there are $m^m - 1$ options for closed-loop system.. The best solution is the one while does not have any unstable eigenvalue, has the smallest phase plane of isolated states.

CASE STUDY: A SYSTEM WITH 3 COLLOCATED ACTUATORS AND SENSORS

A simple lumped longitudinal vibration system has been considered as Figure 1 and the minimum energy eigenstructure assignment using mode orthogonalizing method has been applied in order to isolate the left side of the system from the vibration.

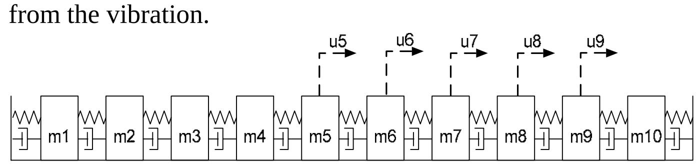


Figure 1. The system of 10 masses with interconnecting springs and dampers.

To avoid the complexity of vibrational modes, a system is chosen to merely consist of the longitudinal vibrations. This system consists of 10 masses, which are interconnected by springs and dampers, as indicated in Figure 1. The goal is to isolate m_{1-4} from an impulse disturbance that is applied on m_{10}. In order to show the effect of the location of the actuators, while not overwhelming the system with too many actuators, the system is considered to have 3 actuators.

All the masses are equal to $50\,kg$ and all the spring coefficients are identical and are equal to $1000\,N/m$. Damping coefficients are assumed to be identical and equal to

10 $N.s/m$. Simulation has been done using Matlab and the control algorithm has been implemented.

The open-loop eigenvalues λ_1, λ_3 and λ_5 have been used to find the associated null spaces. Therefore the problem is finding the appropriate r^1, r^2 and r^3 for $V_{12}^1 r^1$, $V_{12}^2 r^2$ and $V_{12}^3 r^3$, respectively. This system consists of three pairs of collocated actuators and sensors which are located on different masses for each case. Table 1 shows the locations of the actuators for each case and non-zero elements of matrices B and C.

Table 1. Location of actuators for different cases and the non-zero elements of matrices B and C.

Case	Actuator location	Non-zero elements
1	6,7,8	$B(16,1)=B(17,2)=B(18,3)=-1/50$ $C(1,6)=C(2,7)=C(3,8)=1$
2	5,7,8	$B(15,1)=B(17,2)=B(18,3)=-1/50$ $C(1,5)=C(2,7)=C(3,8)=1$
3	6,7,9	$B(16,1)=B(17,2)=B(19,3)=-1/50$ $C(1,6)=C(2,7)=C(3,9)=1$
4	5,6,8	$B(15,1)=B(16,2)=B(18,3)=-1/50$ $C(1,5)=C(2,6)=C(3,8)=1$
5	6,8,9	$B(16,1)=B(18,2)=B(19,3)=-1/50$ $C(1,6)=C(2,8)=C(3,9)=1$
6	6,8,9	$B(16,1)=B(18,2)=B(19,3)=-1/50$ $C(1,6)=C(2,8)=C(3,9)=1$ $C(1,7)=C(2,7)=C(3,7)=1$

Figure 2 shows the displacements of m_1 for each case due to an impulse input applied to m_{10}. Also Figure 3 shows the actuation forces for inner, middle and outer actuators. The outer actuators are the one which is closer to the inputs. Table 2 shows the maximum amplitude of vibration, the reduction percentage of the amplitude of m_1, maximum actuation forces and the work that each actuator has done.

Comparison of the results of the cases 1-4, states that case 3 shows a better result when the maximum force and the energy consumed by actuators are important. Case 3 shows considerable vibration suppression too. If the goal is the reduction of amplitude of vibration, case 4 is the best option. As a conclusion, if the actuators are closer to the input, the maximum force and energy consumption will reduce. On the contrary, if the actuators are closer to the isolated area, the vibration suppression is more significant.

Table 2. Comparison of different cases

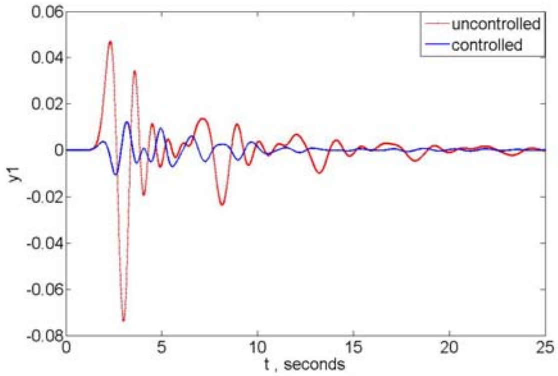
Case number	mass number for actuator location	maximum displacement of m1(mm)	reduction percentage	maximum force of inner actuator (N)	maximum force of middle actuator (N)	maximum force of outer actuator (N)	energy applied by inner actuator (J)	energy applied by middle actuator (J)	energy applied by outer actuator (J)
1	6,7,8	12.2	-73.9	9.8	111.2	60.9	3.9	50.1	54.7
2	5,7,8	5.1	-89.1	8.4	108.8	72.4	0.23	3.5	20.8
3	6,7,9	5.2	-88.9	47.8	72.2	50.7	1.6	13.5	10.7
4	5,6,8	2	-95.7	51.3	99	88.2	1.5	14.8	31.7
5	6,8,9	8.2	-82.5	21.24	106.6	101.8	1.8	8.9	31.7
6	6,8,9	9.5	-79.7	98.2	35.3	99.1	5.3	8.6	42

When the actuators are located on m_6, m_8 and m_9, the system shows instability for all the possible closed-loop solutions. This happens because one of the actuators has been located on a node of one of the closed-loop modes of vibration. To eliminate the instability, two different methods have been used. The first method is to include the eigenvectors associated to unity eigenvalues of $[V_{12}^i]^*[V_{12}^i]$. This leads to a gain matrix that is not full rank. As a result two of the actuators apply similar forces to the system, as can be seen in case 5. The other method is to change C matrix in such a way that the state associated to m_7 is included in the output matrix as done in case 6.

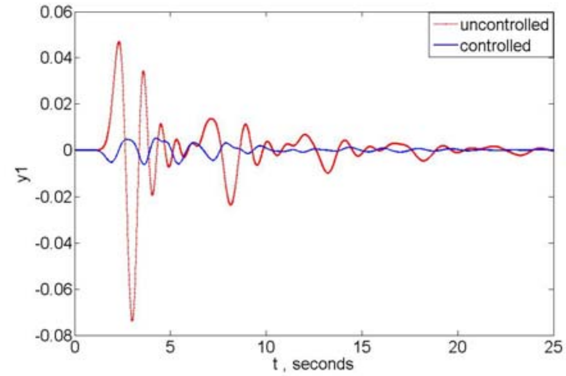
Case 5 is an example that the minimum modal energy algorithm can be extended to consider the eigenvectors associated with the unity eigenvalues of $[V_{12}^i]^*[V_{12}^i]$. By excluding the regenerated open-loop system, the number of generated closed-loop can be considered as $m^m - 1$. This can be a solution for the systems that the actuators are restricted to be placed on the areas that may result in unstable closed-loop system.

CONCLUSION

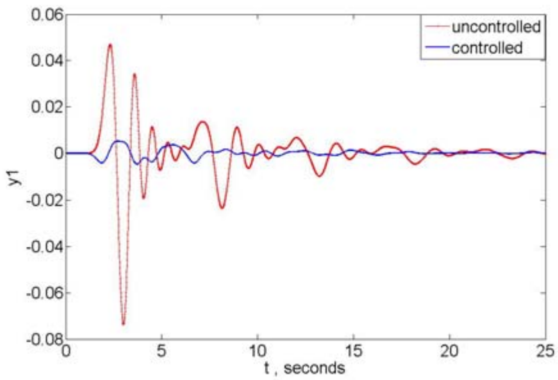
A new eigenstructure assignment method which minimizes the modal energy of the closed-loop system has been introduced here. It is an output feedback control and can be applied to the linear systems with collocated actuators and sensors. This method uses the singular value decomposition to find the null space of the eigenvectors of the closed-loop system.



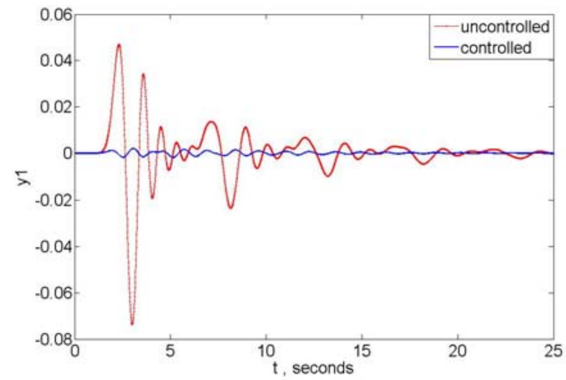
Case 1, actuators on m_6 , m_7 and m_8 .



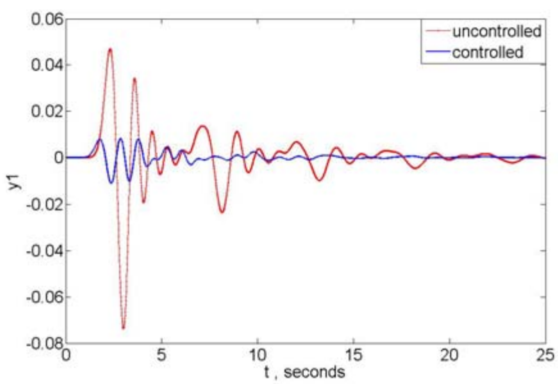
Case 2, actuators on m_5 , m_7 and m_8 .



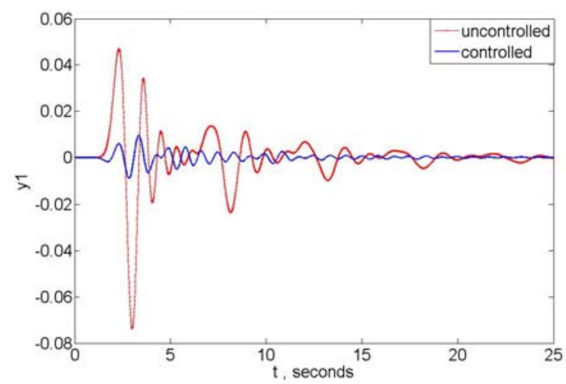
Case 3, actuators on m_6 , m_7 and m_9 .



Case 4, actuators on m_5 , m_6 and m_8 .

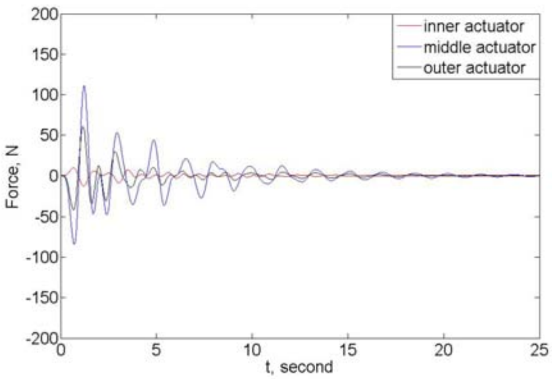


Case 5, actuators on m_6 , m_8 and m_9 .

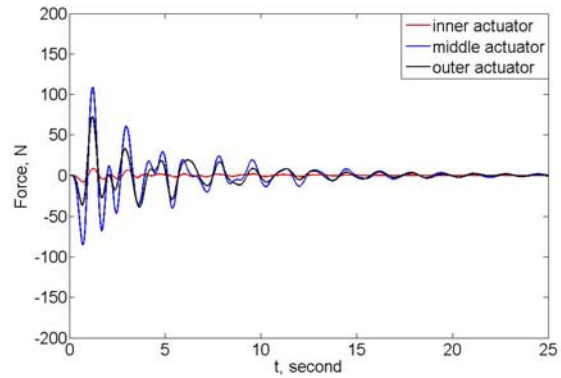


Case 6, actuators on m_6 , m_8 and m_9 .

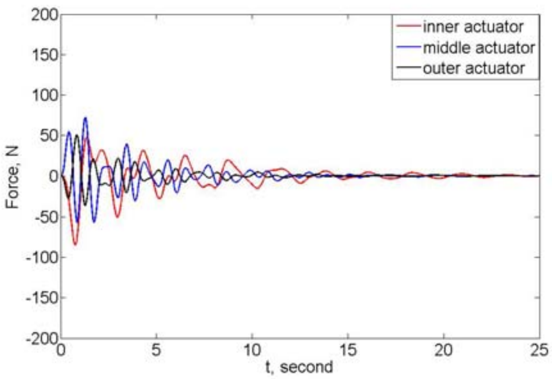
Figure 2. Displacement of m_1 for open-loop and closed-loop systems due to an impulse input on m_{10}



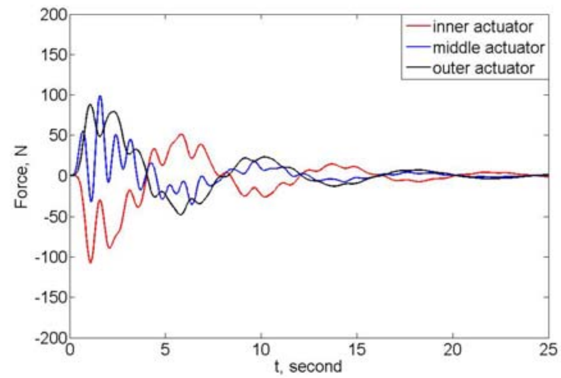
Case 1, actuators on m_6 , m_7 and m_8 .



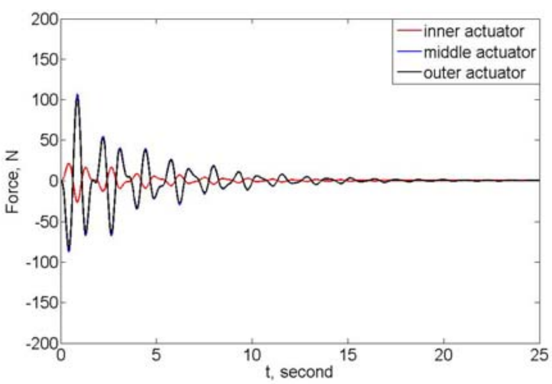
Case 2, actuators on m_5 , m_7 and m_8 .



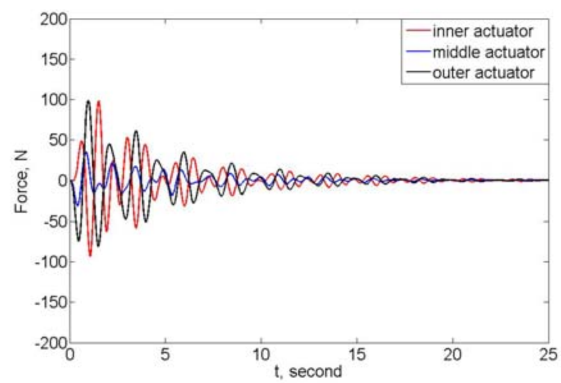
Case 3, actuators on m_6 , m_7 and m_9 .



Case 4, actuators on m_5 , m_6 and m_8 .



Case 5, actuators on m_6 , m_8 and m_9 .



Case 6, actuators on m_6 , m_8 and m_9 .

Figure 3. Actuation forces at different locations for closed-loop systems due to an impulse input on m_{10}

Minimum modal energy ESA is based on developing a procedure for regenerating the open-loop system. Using the regenerated open-loop system, closed-loop system can be generated which has orthogonal eigenvectors to the open-loop ones. Conventional ESA methods require defining a desirable eigenvectors. Since there is no guaranty that the desirable eigenvectors lay within the admissible sub-space of the eigenvectors, an error is unavoidable. Minimum modal energy ESA does not need defining the desirable eigenvectors. In fact, this method finds the eigenvectors within admissible eigenvector subspace which are orthogonal to the open loop eigenvectors, which means the isolation is not depended on the type of the disturbance. Moreover, this new method does not specify a location for the closed-loop eigenvalues; therefore the actuation forces can be reduced. In summary this method does not need any prior prediction about the closed-loop system behavior and the results of the identification of the open-loop systems are the only data that are used in the design of the control law.

It has been shown that the vibration of the isolated area has been reduced. If the maximum force and the consumption of energy by actuators are the criteria, the actuators need to be closer to the disturbance sources. But, if the reduction of the amplitude of vibration is the goal, actuators need to be closer to the isolated area. Also it has been shown that the minimum modal energy eigenstructure assignment method can be extended to $m^m - 1$ possible solutions while m is the number of actuators.

REFERENCES

[1] B.C. Moore, On the Flexibility Offered by State Feedback in Multivariable Systems Beyond Closed Loop Eigenvalue Assignment, IEEE Transactions on Automatic Control (1976) 689-692.

[2] W.W. Clark, F.J. Shelley, Experiments in Eigenstructure Assignment for Active Mode Localization in A Flexible Beam, American Control Conference, Albuquerque, New Mexico, 1997, pp. 1859-1863.

[3] T.B. Cunningham, Eigenspace Selection Procedures for Closed Loop Response Shaping with Modal Control, Proc. of the 19th IEEE Conference on Decision and Control, Albuquerque, New Mexico, 1980, pp. 178-186.

[4] S. Choura, A.S. Yigit, Confinement and Suppression of Structural Vibrations, Transactions of the ASME 123 (2001) 496-501.

[5] F.J. Shelley, W.W. Clark, Eigenvector Scaling for Mode Localization in Vibrating Systems, Journal of Guidance, Control, and Dynamics 19 (1996) 1342-1348.

[6] L.R. Corr, W.W. Clark, Vibration Confinement Using Piezoelectric Transducers and Eigenstructure Placement, AIAA-99-1552 (1999).

[7] L.R. Corr, W.W. Clark, Active and Passive Vibration Confinement using Piezoelectric Transducers and Dynamic Vibration Absorbers, SPIE Conference on Smart Structures and Integrated Systems, Newport Beach, California, 1999, pp. 747-758.

[8] F.J. Shelley, W.W. Clark, Experimental Application of Feedback Control to Localize Vibration, Journal of Vibration and Acoustics 122 (2000) 143-150.

[9] F.J. Shelley, W.W. Clark, Active Mode Localization in Distributed Parameter Systems with Consideration of Limited Actuator Placement, Part 2: Simulations and Experiments, Journal of Vibration and Acoustics 122 (2000) 165-168.

[10] F.J. Shelley, W.W. Clark, Active Mode Localization in Distributed Parameter Systems with Consideration of Limited Actuator Placement, Part 1: Theory, Journal of Vibration and Acoustics 122 (2000) 160-164.

[11] J. Tang, K.W. Wang, A Simultaneous Active-Passive Approach for Structural Vibration Confinement Using Piezoelectric Actuators, 44th AIAA/ASME/ ASCE/AHS Structures, Structural Dynamics, and Materials Conferences, Norfolk, Virginia, 2003, pp. 1-11.

[12] J. Tang, K.W. Wang, Vibration Confinement via Optimal Eigenvector Assignment and Piezoelectric Networks, Journal of Vibration and Acoustics 126 (2004) 27-36.

[13] T.Y. Wu, K.W. Wang, Vibration Isolator Design via Energy Confinement through Eigenvector Assignment and Piezoelectric Networking, SPIE, Smart Structures and Materials: Damping and Isolation, 2004, pp. 11-25.

[14] G.L. Slater, Q. Zhang, Controller Design by Eigenspace Assignment, AIAA-90-1193-CP (1990) 19-31.

[15] G.H. Golub, C.F.V. Loan, Matrix Computations, third ed., The Johns Hopkins University Press, 1996.