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MATHEMATICAL MODELING OF THERMAL EFFECTS IN STEADY STATE DYNAMICS OF MICRORESONATORS USING LORENTZIAN FUNCTION: PART 2 - TEMPERATURE RELAXATION

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ABSTRACT

Thermal phenomena have two distinct effects, which are called, in this report, “thermal damping” and “temperature relaxation”. In this second part of a two-part report we (only) model and investigate the temperature relaxation and its effects on microresonator dynamics. A reduced order mathematical model of the system is introduced as a mass-spring-damper system actuated by a linearized electrostatic force.

Temperature relaxation is the thermal stiffness softening and is modeled as a decrease in stiffness rate, utilizing a Lorentzian function of excitation frequency. The steady state frequency-amplitude dependency of the system will be derived utilizing averaging perturbation method. Analytic equation describing the frequency response of the system near resonance which can be utilized to explain the dynamics of the system, as well as design of involved dynamic parameters is developed.

1. INTRODUCTION

The aim of this paper is to introduce mathematical modeling and investigate the effect of temperature relaxation (without considering thermal damping) in dynamic behavior and sensitivity analysis of microresonators dynamics. Temperature dependent properties of the microbeam material play a significant role in affecting the design and application of micro systems utilizing a microbeam or microcantilever resonator (Karami and Garnich 2005). Stiffness and damping rates of the microbeam are the most important material characteristics in vibration behavior of the microresonator affected by temperature change.

To investigate the developed mathematical model for the temperature relaxation phenomenon and to analyze its effects on microresonator dynamics, the model is applied to a forced linear vibrating system. Then, a linearized model of electric actuated microbeam resonator will be employed and the effect of temperature relaxation phenomenon is modeled as a decrease in stiffness rate obeying Lorentzian function of excitation frequency. The steady state frequency-amplitude dependency of the system will be derived utilizing averaging perturbation method. The developed analytic equation describing the frequency response of the system around resonance can be utilized to explain the dynamics of the system, as well as resonant frequency and peak amplitude.

Designing MEMS devices are sometimes based on trial and error because most MEMS are modeled by simplified analytical tools, resulting in a relatively approximate prediction of performance behavior. Therefore, micro system design process requires several iterations before the desired performance are finally achieved (Younis, Abdel-Rahman, and Nayfeh 2003, Younis 2004). The reduced-order models, on the other hand, need to be improved as a basis for prediction and optimization tool of the proposed behavior. Reduced-order models have shown their effectiveness in research and design, and are developed to capture the most significant characteristics of a MEMS behavior in a few variables (Younis 2004, Nayfeh and Younis 2004).

Most electric actuated microbeam-based resonators, sensors and actuators must work at resonance. Typical microresonator devices are made by a parallel capacitor, in which one electrode is fixed and the other is allowed to move using some flexibility. The movable electrode, fabricated in the form of microbeam, microplate, or microcantilever, serves as a mechanical resonator. It is actuated electrically and its motion can be detected by capacitive changes. This motion of the movable electrode can be converted to an electric signal in the capacitance, which is related to the physical quantity being measured (Younis, and Nayfeh 2003).

2. MATHEMATICAL MODELING OF TEMPERATURE RELAXATION

Thermoelastic phenomenon affects the rigidity of the material, since the rigidity is a temperature dependent characteristic. Most engineering materials become softer at higher temperatures. Heat flows to attempt to restore equilibrium, causing the restoring force from the microbeam to relax from its initial value to a smaller equilibrium value (Barmatz and Chen 1974; Saulson 1990; Gysin et al, 2004). Temperature relaxation is a consequence of the microbeam being in thermal equilibrium with its environment. Temperature relaxation depends on the thermodynamic properties of the material which are functions of temperature. Temperature relaxation is proportional to frequency; hence, when the principal natural frequency increase while the size of devices decreases, the Temperature relaxation becomes more significant (Lifshitz and Roukes 2000).

Since the warming of the microbeam material is Lorentzian frequency dependent, the effect of stiffness softening of the microbeam is also a frequency dependent characteristic. So, we present a negative softening function to define this behavior. More specifically, a negative restoring force with stiffness as a Lorentzian function of excitation frequency

$$f_{Ts} = -k_T \frac{\omega/\omega_1}{1+(\omega/\omega_1)^2} w \tag{1}$$

determines the drop in linear rigidity stiffness force, $f_r = EI\left(\partial^4 w / \partial x^4\right)$. The breaking frequency of the thermal stiffness softening is also at the fundamental resonance frequency. The softening stiffness coefficient per unit length, k_T, must be determined experimentally.

Zener was the first researcher who investigated and modeled the effect of internal frictions in resonating thermoelastic solids known as thermoelstic effects (Zener 1937, 1938a, 1938b, 1947). Lorentzian function which is shown in Figure 1 has been derived and used in thermal effects analysis, especially in flexural beam resonators by most of researchers (Srikar and Senturia 2002, Yang et al, 2002; Abdolvand et al, 2003; Jeong et al, 2003; De and Aluru 2004; Fejer at al, 2004; Husman et al, 2004).

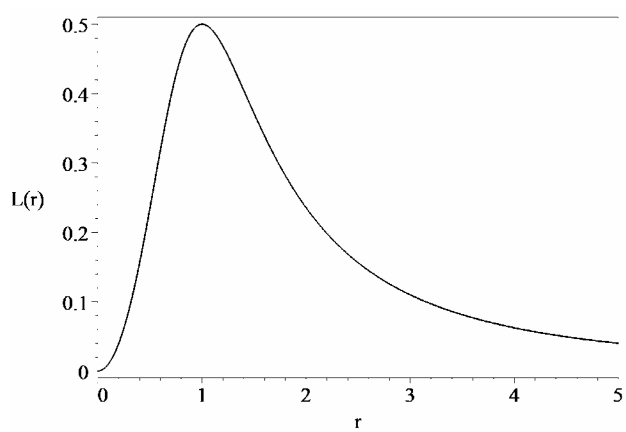


Figure 1. Lorentzian function

For a linear mass-spring-dashpot oscillator, we introduce a frequency-dependent force

$$f_{Ts} = -k_T \; \mathcal{L}\left(\frac{\omega^2}{\omega_1^2}\right)\dot{z} \tag{2}$$

to simulate the stiffness softening corresponding to temperature relaxation. The coefficient k_T defines the thermal stiffness softening per unit length of the microbeam, which depends on geometric parameters and material properties of the microbeam must be determined experimentally. Thermal stiffness softening force introduces a linear spring with frequency dependent rate, which is maximum at fundamental resonance frequency. More specifically temperature relaxation effect is modeled by a negative stiffness force with a Lorentzian frequency-dependent coefficient.

Employing (2) indicates the damping exhibit a broad maximum at natural frequency ω_1. This is in agreement with the classic phenomenon called anelasticity (Zener 1947; Saulson 1990).

3. THERMAL DAMPING EFFECT IN SINGLE DOF VIBRATING SYSTEMS

Figure 2 depicts a vibration isolation system. The base is excited by a harmonic displacement $y = Y\,sin(\omega t)$ where Y is the amplitude and ω is the frequency of the excitation. Overall stiffness of the system is a combination of linear stiffness k and the thermal stiffness softening $-k_T\,(\omega/\omega_1)^2 / \left(1+(\omega/\omega_1)^4\right)$.

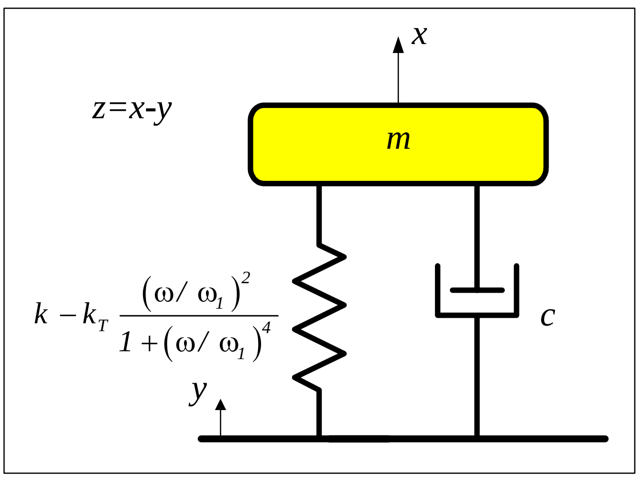


Figure 2. Lorentzian function

The equation of motion of the system is

$$m\ddot{x} + c\left(\dot{x} - \dot{y}\right) + \left(k - k_T \frac{(\omega/\omega_1)^2}{1+(\omega/\omega_1)^4}\right)(x-y) = 0 \quad (3)$$

After introduction of a relative displacement coordinate z=x-y, and employing a set of dimensionless parameters, this equation transforms to the following equation,

$$u'' + a_1 u' + \left(1 - a_3 \frac{r^2}{1+r^4}\right) u = r^2\, sin(r\tau) \quad (4)$$

where

$$\begin{array}{llll} x - y = z & \dfrac{z}{Y} = u & \omega_1 = \sqrt{\dfrac{k_1}{m}} & \omega_1 t = \tau \\ r = \dfrac{\omega}{\omega_1} & a_1 = \dfrac{c}{\sqrt{k_1 m}} & a_3 = \dfrac{k_T}{\sqrt{k_1 m}} & \end{array} \quad . \quad (5)$$

It can be shown that the steady state solution of (4) is

$$\begin{aligned} U = r^2\left(1+r^4\right)\Big[& r^{12} + \left(a_1^2 - 2\right) r^{10} + \left(2a_3 - 3\right) r^8 \\ & + \left(2a_1^2 + 2a_3 - 4\right) r^6 \left(a_3^2 + 2a_3 + 3\right) r^4 \\ & + \left(a_1^2 - 2a_3 - 2\right) r^2 + 1 \Big]^{-1/2} \end{aligned} \quad (6)$$

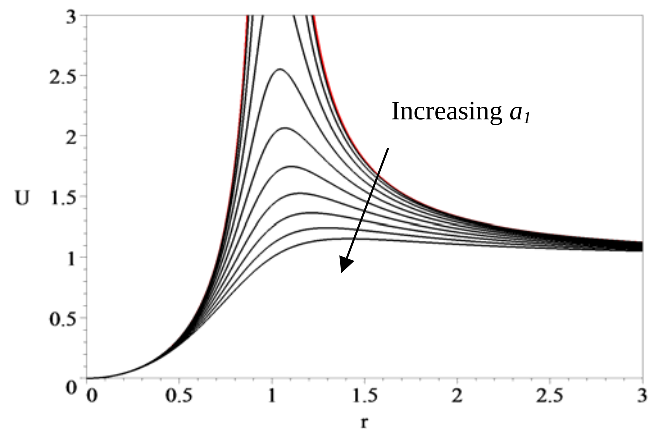


Figure 3. U for $a_3=0$ and $a_1=0,0.1,\ldots,1.0$.

Figures 3 and 4 show the frequency response of the system without and with temperature relaxation, respectively. More specifically, in Figure 3, $a_3=0$, and in Figure 4, $a_3=0.1$. The value of the parameters of different curves, a_1, starts from zero and goes up to one, with an increment of 0.1. Increasing a_1 decreases the amplitude in both figures as expected. Because of small value of a_3 the difference between the figures is not obvious. To clarify the difference, with $a_1=0.4$, U was plotted for $a_3=0$ and $a_3=0.1$ in Figure 5. The upper curve corresponds to no temperature relaxation while the lower curve is for $a_3=0.1$. Figure 5 clearly shows that the effect of temperature relaxation is ignorable for off resonance; however it has maximum effect at resonant frequency. Note that temperature relaxation changes the resonance frequency and affects the predicted dynamics of microresonator when the temperature relaxation is not simulated.

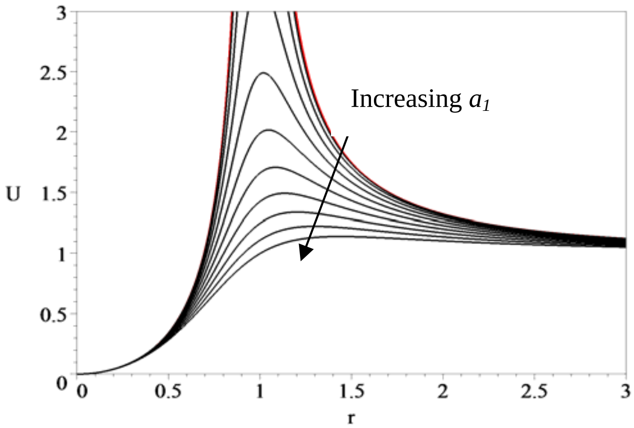


Figure 4. U for $a_3=0.1$ and $a_1=0,0.1,\ldots,1.0$.

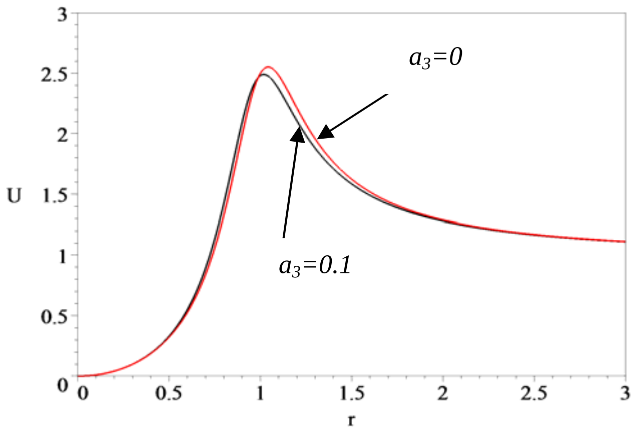


Figure 5. U for $a_1=0.4$ and $a_3=0, 0.1$.

4. REDUCED ORDER MODEL OF MICRORESONATORS

A typical microresonator is composed of a microcantilever resonator attached to a microplate which is the moving electrode of a variable capacitor. There is a ground plane underneath the beam. A DC-bias voltage, v_p, is applied to the resonator while an AC excitation voltage is applied to its underlying ground plane. A simplified mechanical model of the system is illustrated in Figure 6. A voltage difference between opposite electrodes of the variable capacitor acts as an electric load actuation. The capacitor deforms under the induced electrostatic force until the electrostatic force is balanced by the restoring mechanical forces. The electric load is a combination of the DC polarization voltage, v_p, and the harmonic AC actuating voltage, $v = v_i \sin(\omega t)$. The polarization voltage may be high enough to collapse the system and make a short circuit between electrodes. The minimum polarization voltage to flex the microcantilever to contact the fixed electrode is called "collapse" load. Beyond the collapse load the mechanical restoring force can not resist its opposing electrostatic force (Hsu 2002).

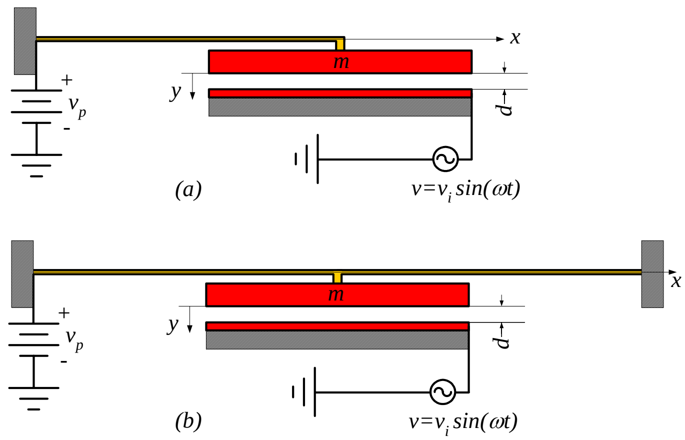


Figure 6. A microcantilever and a clamed-clamped microbeam model of microresonators.

The one dimensional electrostatic force, f_e, between two electrodes is

$$f_e = \frac{\varepsilon_0 A (v - v_p)^2}{2(d - w_0)^2}, \qquad v = v_i \sin(\omega t) \qquad (7)$$

where, $\varepsilon_0 = 8.85 \times 10^{-12}\ As/Vm$ is permittivity in vacuum, A is the effective area of the microplate, and $w=w(x,t)$ is the lateral displacement of the microbeam (Hsu 2002).

Excitation of the microbeam with a frequency close to the fundamental resonance frequency of the beam causes the resonator to start oscillation. Oscillation of the microbeam creates a time varying capacitance $C = \varepsilon_0 A / (d - w)$. Uniform electric load across the electrodes needs a long microbeam and a short microplate electrode. Under this assumption, variation of the beam deflection across the length of the electrodes is ignorable (Nguyen 1995).

When the beam's geometry is uniform, lateral vibrations of the microbeam can be described by the following equation.

$$\rho\frac{\partial^2 w}{\partial t^2}+c\frac{\partial w}{\partial t}+EI\frac{\partial^4 w}{\partial x^4}-k_T\frac{\left(\omega/\omega_1\right)^2}{1+\left(\omega/\omega_1\right)^4}w =\frac{\varepsilon_0 A\left(v-v_p\right)^2}{2\left(d-w_0\right)^2} \quad (8)$$

The following dimensionless parameters are defined to make the equation of motion dimensionless. The parameter n is a constant depending on mode shape of the microbeam.

$$\tau=\omega_1 t \qquad \omega_1=\frac{n^2}{L^2}\sqrt{\frac{EI}{\rho}} \qquad z=\frac{x}{L}$$
$$y=\frac{w}{d} \qquad Y=\frac{w_0}{d} \qquad r=\frac{\omega}{\omega_1} \qquad a_1=\frac{cL^2}{n\sqrt{\rho EI}} \quad (9)$$
$$a3=\frac{k_T L^4}{n^2 EI} \qquad a_4=\frac{\varepsilon_0 A L^4}{2n^2 d^3 EI}$$

Therefore, the lateral vibration of the microbeam of the resonator system reduces to the following PDE.

$$\frac{\partial^2 y}{\partial\tau^2}+a_1\frac{\partial y}{\partial\tau}+\frac{\partial^4 y}{\partial z^4}+a_3\frac{r^2}{1+r^4}y=a_4\frac{\left(v-v_p\right)^2}{\left(1-Y\right)^2} \quad (10)$$

Assuming a separation of variable

$$y=Y(\tau)\cdot\varphi(z) \quad (11)$$

where the mode shape function $\varphi(z)$ is determined by satisfying boundary conditions. We accept a first harmonic shape function to reduce the PDE (10) to an ordinary differential equation for the temporal function $Y(\tau)$ that represent the maximum deflection of the microbeam. The maximum deflection occurs at the tip of microcantilever.

Mode shape for a microcantilever must satisfy the following boundary conditions.

$$y\left(0,\tau\right)=0 \qquad \frac{\partial}{\partial z}y\left(0,\tau\right)$$
$$\frac{\partial^2}{\partial z^2}y\left(1,\tau\right)=0 \qquad \frac{\partial^3}{\partial z^3}y\left(1,\tau\right)=0 \quad (12)$$

The first harmonic mode shape satisfying the required boundary conditions, is

$$\varphi(x)=\cos\left(\frac{\pi x}{2}\right). \quad (13)$$

and the mode shape parameter is

$$n=\frac{\pi^2}{4}. \quad (14)$$

Therefore, the differential equation for the temporal function $Y(\tau)$ reduces to

$$\ddot{Y}+h\dot{Y}+\left(1-a_3\frac{r^2}{1+r^4}\right)Y=\frac{1}{\left(1-Y\right)^2}\times\left[\left(\alpha+\beta\right)+2\sqrt{2\alpha\beta}\sin\left(r\tau\right)-\beta\cos\left(2r\tau\right)\right] \quad (15)$$

where,

$$h=a_1 \quad \alpha=a_4 v_P^2 \quad 2\sqrt{2\alpha\beta}=2a_4 v_P v_i \quad \beta=\frac{a_4}{2}v_i^2. \quad (16)$$

When the amplitude of the lateral vibration is too small $Y<<1$, the governing equation of the microresonator may be linearized to examine the temperature relaxation,

$$\ddot{Y}+h\dot{Y}+\left(1-a_3\frac{r^2}{1+r^4}\right)Y +2\left(\frac{1}{2}-\beta-\alpha+\beta\cos\left(2r\tau\right)-2\sqrt{2\alpha\beta}\sin\left(2r\tau\right)\right)Y =\alpha+2\beta\sin^2\left(r\tau\right)+2\sqrt{2\alpha\beta}\sin\left(r\tau\right) \quad (17)$$

Due to polarization voltage and temperature relaxation, the equilibrium position of the system changes to

$$Y_0=\frac{\alpha}{1-a_3\frac{r^2}{1+r^4}-2\alpha}. \quad (18)$$

5. TEMPERATURE RELAXATION EFFECT IN MICRORESONATORS

Because the equilibrium of the microresonator is affected by dynamic parameters of the system, the following approximate solution must be utilized to apply the multiple time scale perturbation method and determine the amplitude of oscillation around resonance

$$y=A_0+A(\tau)\sin\left(r\tau+\psi(\tau)\right) \quad (19)$$

$$y'=A(\tau)r\cos\left(r\tau+\psi(\tau)\right) \quad (20)$$

Substituting (19) and (20) in (17) and eliminating secular terms A_0 is obtained as:

$$A_0 = \frac{\alpha + \beta}{1 - 2\alpha - 2\beta - a_3 \dfrac{r^2}{1 + r^4}}. \tag{21}$$

Applying multiple time scale method produces the following equation describing the steady state amplitude of oscillation as a function of excitation frequency affected by dynamic parameters of the system implicitly.

$$\begin{aligned} &A^2 r^{12} + Z_1 r^{10} + \left(Z_2 - Z_3\sqrt{Z_4}\right) r^8 + Z_5 r^6 \\ &+\left(Z_6 - Z_7\sqrt{Z_4}\right) r^4 + Z_8 r^2 \\ &+Z_9 - Z_3\sqrt{Z_4} = 0 \end{aligned} \tag{22}$$

where

$$Z_1 = A^2\left(4\beta - 2 + 4\alpha + h^2\right) \tag{23}$$

$$\begin{aligned} Z_2 = &\left(2a_3 - 4\beta + 3\beta^2 + 4\alpha^2 - 4\alpha + 8\alpha\beta + 3\right)A^2 \\ &-4\alpha\beta\left(2A_0^2 + 1\right)^2 \end{aligned} \tag{24}$$

$$Z_3 = 4\sqrt{\alpha\beta}\left(2A_0 + 1\right) \tag{25}$$

$$\begin{aligned} &Z_4 = A^2 r^2\left(r^4 + \left(\beta + 2\alpha - 1\right)r^2 + \left(1 + a_3\right)\right) \\ &+A^2\left(\beta + 2\alpha - 1\right) - \left(4A_0^2\alpha\left(1 + A_0\right) + 1\right)\alpha\left(1 + r^4\right) \end{aligned} \tag{26}$$

$$Z_5 = A^2\left(2h^2 + \left(2a_3 + 2\right)\left(2\alpha - 1 + 2\beta\right)\right) \tag{27}$$

$$\begin{aligned} Z_6 = &\left(a_3\left(a_3 + 2\right) + 6\beta^2 + 8\alpha^2 + 16\alpha\beta + 3\right)A^2 \\ &-32\alpha\beta A_0\left(A_0 + 1\right) - 8\alpha\beta \end{aligned} \tag{28}$$

$$Z_7 = 2Z_3 \tag{29}$$

$$Z_8 = \left(2a_3\left(2\alpha + 1 + 2\beta\right) + 4\beta + 4\alpha - 2 + h^2\right)A^2 \tag{30}$$

$$\begin{aligned} Z_9 = &\left(3\beta^2 + 4\alpha^2 + 8\alpha\beta + 1 - 4\alpha - 4\beta\right)A^2 \\ &-16\alpha\beta A_0\left(A_0 + 1\right) - 4\alpha\beta \end{aligned} \tag{31}$$

6. DYNAMIC ANALYSIS

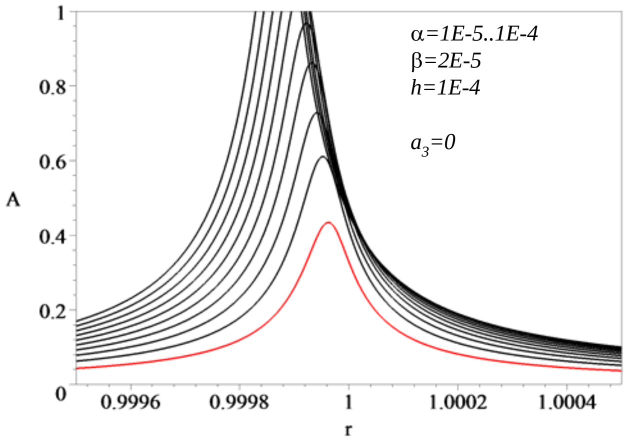
Polarization voltage parameter α, alternative excitation voltage parameters β, damping parameter h, the excitation frequency ratio r, as well as the temperature relaxation parameters a_3 affect the frequency behavior of the microresonator described by Equation (22). To determine the dependency of steady state behavior of the microresonator to the dynamical parameters involved, we graphically illustrate the frequency response for various parameters utilizing Table 1. Table 1 introduces the nominal values of a sample micrcantilever resonator (Kanda et al, 2000; Yang et al 2002; Khaled et al, 2003; Kaajakari et al, 2004).

Table 1. Nominal parameters of the microresonator.

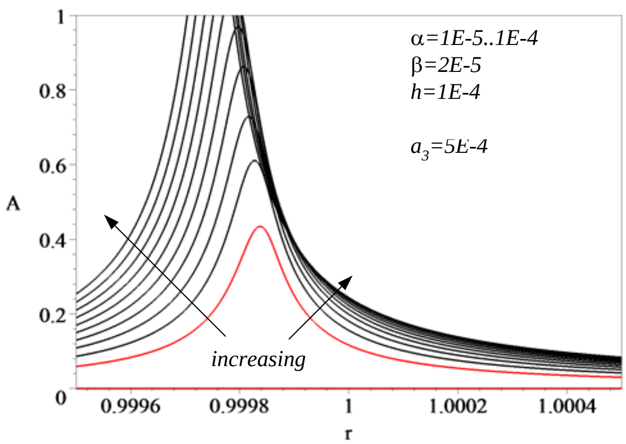
m	1×10^{-11} kg
c	1×10^{-8} Ns / m
k	1 N/m
d	2.0 μm
α	$0.0000553125 v_p^2$
A	200×50 μm
β	$0.00002765625 v_i^2$

Effect of variation of polarization voltage for a set of parameters is depicted in Figure 7. It is seen that the amplitude of steady state oscillation increases by increasing the polarization voltage. No temperature relaxation is shown in Figure 7(a) where a_3 is set to zero. Temperature relaxation decreases the stiffness of the system near resonance, and hence decreases the resonance frequency much more than increasing the polarization voltage (Figure 7(b)). The value of thermal softening is set to around 20% of the nominal value of linear stiffness of the system. The peak value of the frequency response at resonance is a monotonically increasing function of the polarization voltage.

The effect of variation of excitation voltage is similar to the effects of alteration of the polarization voltage. More specifically, the peak value increases and the resonant frequency shifts to lower frequencies when the amplitude of the excitation voltage increases. There is also a higher limit for both polarization and alternative voltages to have oscillation within the gap size limit. Variation of excitation voltage is shown in Figure 8.



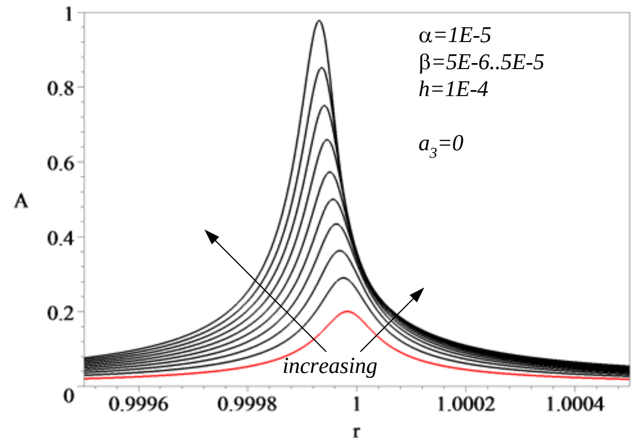
(a)



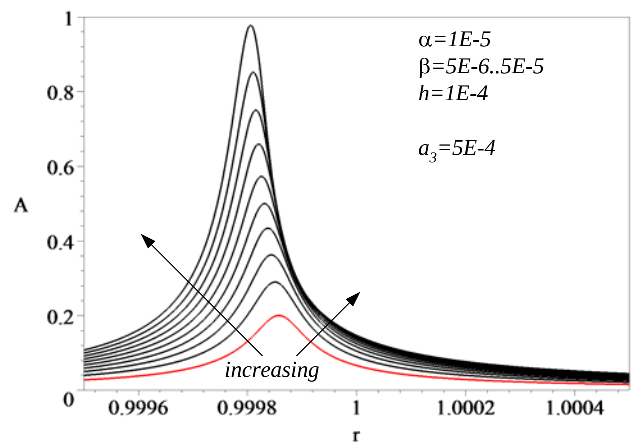
(b)

Figure 7. Effect of variation of polarization voltage.

The most sensitive and important effect of this relaxation is the shift of resonance frequency. Shift is a very important phenomenon in microreonator dynamics, simply because shift of resonant frequency is what we measure when microresonators are being utilized as sensors. The resonance frequency also indirectly affects design parameters (Sudipo and Aluru 2004). To determine the capability of the MEMS to sense a shift in resonance frequency by varying a parameter, Equation (22) must be investigated numerically.



(a)



(b)

Figure 8. Effect of variation of excitation voltage.

Sensitivity of resonance frequency, r_0 , is illustrated in Figures 9 and 10. The value of temperature relaxation coefficients a_3 is set to zero and the results are depicted in Figure 9(a). It is seen that the resonance frequency is a monotonically decreasing function of both polarization and excitation voltages. Behavior of resonant shifting looks linear with variation of both voltages within the limit of possible variation of voltages according to the physical limit of gap. In addition, introducing temperature relaxation maintains the linearity of resonant frequency and excitation voltages relationship, although it shifts down the value of the frequency (Figure 9(b)).

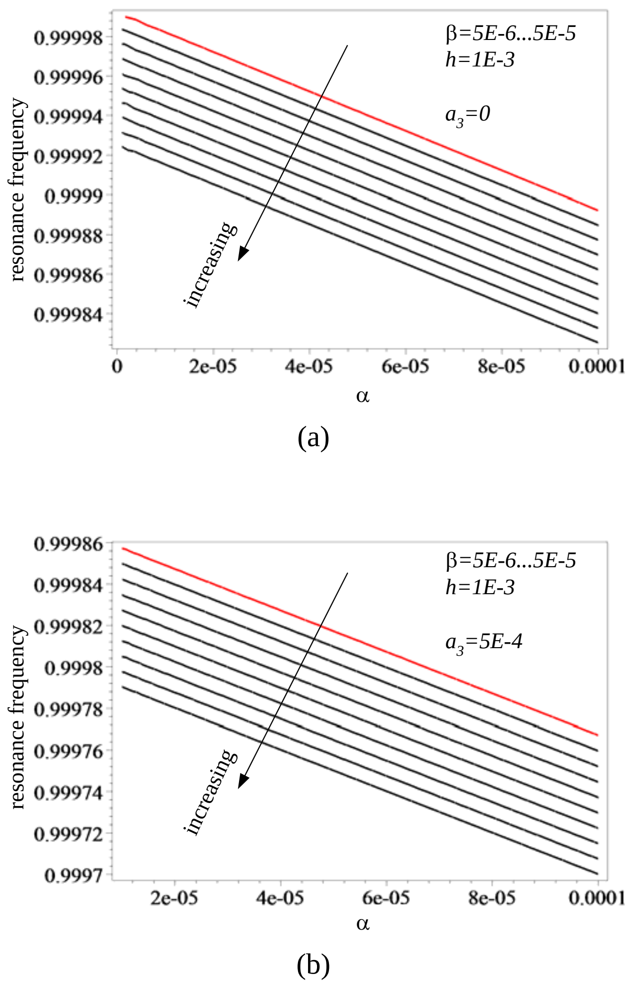


Figure 9. Effect of variation of excitation and polarization voltages on shift of resonance.

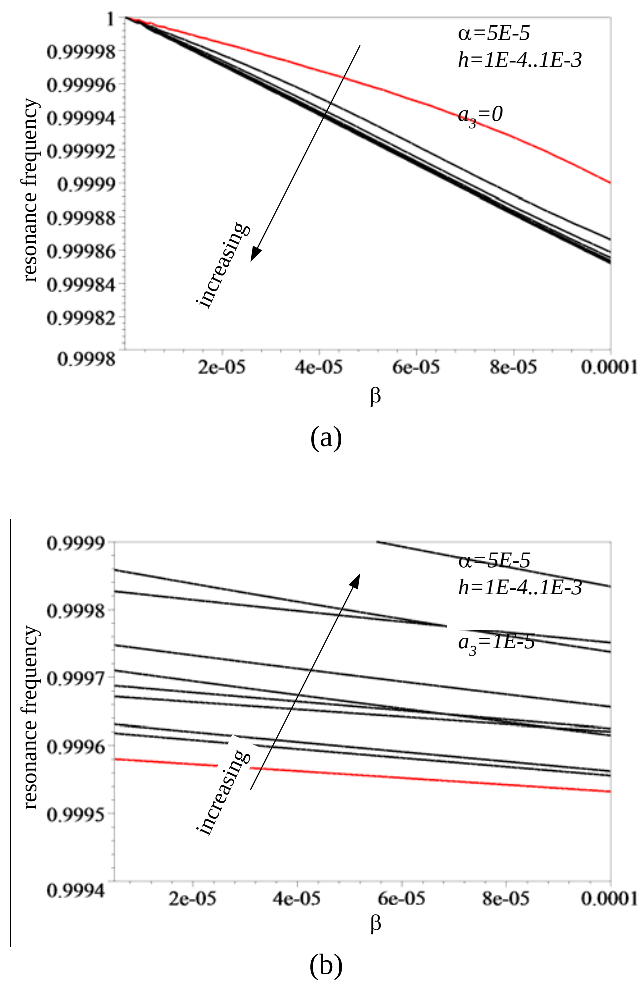


Figure 10. Effect of the variation of damping and excitation voltage on shift of resonance.

Figure 10 depicts the behavior of resonant frequency when damping rate varies. It can be seen that behavior of resonant frequency is neither linear nor monotonic when damping is varied. Figure 10(a) shows the effect of variation of damping and excitation voltage on resonant shift when there is no temperature relaxation. The effect of damping variation when temperature relaxation is present is shown in Figures 10(b).

7. CONCLUSION

We presented a model for temperature relaxation based on Lorentzian function, and analyzed the effects of temperature relaxation in single dof vibrating system as well as elastic actuated microresonator. We defined the temperature relaxation by an effective force per unit length of the vibrating microbeam. Temperature relaxation is due to warming and softening of the microbeam material due to heat energy generated. The temperature relaxation is described by a suitable Lorantzian function of excitation frequency with a maximum at the resonance frequency of the system.

Nondimensionlized equation of flexural motion of the microbeam has been derived including temperature relaxation modeling. Employing multiple time scale method, we derived the amplitude-period relationship at steady state conditions. Then the frequency response of the system has been analyzed and resonant frequency shifting of vibration has been investigated by varying dynamic parameters involved. Temperature relaxation has a significant effect on resonant shifting.

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