Vibration Cancellation in a Plate Using Orthogonal Eigenstructure Control

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Orthogonal eigenstructure control is a novel control method that can be used for vibration suppression in flexible structures. The method described in this study does not need defining the desired locations of the closed-loop poles or predetermining the closed-loop eigenvectors. The method, which is applicable to linear multi-input multi-output systems, determines an output feedback control gain matrix such that some of the closed-loop eigenvectors are orthogonal to the open-loop eigenvectors. Using this, the open-loop system's eigenvectors as well as a group of orthogonal vectors are regenerated based on a matrix that spans the null space of the closed-loop eigenvectors. The gain matrix can be generated automatically; therefore, the method is neither a trial and error process nor an optimization of an index function. A finite element model of a plate is used to study the applicability of the method to systems with relatively large degrees of freedom. The example is also used to discuss the effect of operating eigenvalues on the process of orthogonal eigenstructure control. The importance of the operating eigenvalues and the criteria for selecting them for finding the closed-loop system are also investigated. It is shown that choosing the operating eigenvalues from the open-loop eigenvalues that are farthest from the origin results in convergence of the gain matrix for the admissible closed-loop systems. It is shown that the converged control gain matrix has diagonal elements that are two orders of magnitude larger than the off-diagonal elements, which implies a nearly decoupled control. [DOI: 10.1115/1.4001991]

Keywords: orthogonal eigenstructure control, operating eigenvalues, eigenstructure assignment, vibration reduction, plate

1 Introduction

Eigenstructure assignment in multi-input multi-output systems was introduced by Moore. He showed that there is a class of eigenvectors associated with a distinct set of closed-loop eigenvalues [1,2]. This resulted in the idea of eigenstructure assignment, which is both placing of eigenvalues in desired locations and choosing a set of associated eigenvectors from a group of desirable eigenvectors. The speed of response is determined by the assigned closed-loop eigenvalues, and the shape of the response can be adjusted by the assigned eigenvectors. This gives a considerable freedom for defining a controller.

The first practical eigenstructure assignment application that resulted in a desirable transient response was reported by Cunningham [3]. He used singular value decomposition to define the basis of the achievable eigenvector subspace. He applied this technique in an output feedback control method by optimally combining the basis vectors to minimize the error between achievable and desirable eigenvectors. Using singular value decomposition, a finite number of actuators were used for shaping the eigenvectors of the closed-loop system [4].

Shelley and Clark showed that it is not possible to tell how the absolute displacements in a system are changed just by adjusting the system's eigenvectors [5]. They proposed a mode localization technique called eigenvector scaling while studying the time domain response of the system. This method changes specific elements of each eigenvector in order to uniformly decrease the relative displacement of the corresponding areas in the system [6]. They showed analytically that absolute displacements in isolated areas can be reduced by eigenvector shaping, regardless of the type of the disturbance. They also introduced the eigenstructure shaping method as an active control method. This method scales and reforms part of or the entire system mode shapes and regenerates the behavior of the system. Since all the shape modes are scaled in the same way, vibration confinement of the system is not affected by the type of disturbance [2,7–9]. Eigenvector shaping using singular value decomposition has been introduced and used as a solution to the problem of limited actuators/sensors [7]. This method uses a Moore–Penrose generalized left inverse and produces the closest eigenvector in a least squares sense to the desired ones since it gives the minimum Euclidean 2-norm error.

Tang and Wang [8,9] proposed a method that finds optimal eigenvectors using the Rayleigh principle by minimizing the ratio of the modal energy at the concerned area to the modal energy of the whole structure using an auxiliary eigenvalue problem. They showed that predefining the desired eigenvector components can cause unsatisfactory performance if a match between components of the desired and achievable eigenvectors happens in the noncritical degrees of freedom. A case study of this method has been presented in Ref. [10].

Slater and Zhang [11] showed that when the eigenvectors are the only parameters that are changed, the control effort is not necessarily minimized if the closed-loop eigenvalues are forced to be close to the open-loop eigenvalues. A large change in eigenvectors may need a large movement of the eigenvalues to minimize the feedback gains. They also showed that closed-loop eigenvalues and eigenvectors have to be consistent in order to avoid the large control efforts. Also, they proposed that since—at the time—there was no method for having consistent closed-loop eigenvectors and eigenvalues, a minimum number of constraints should be imposed on the elements of the eigenvectors in order to have a readily achievable control effort.

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Orthogonal eigenstructure control (OEC) that has been proposed by Rastgaar et al. [12–15] addresses some of the shortcomings of the eigenstructure assignment methods. Particularly, OEC does not require specification of the locations of the closed-loop eigenvalues. The closed-loop system has eigenvalues that are different from the open-loop eigenvalues, yet are consistent with the closed-loop eigenvectors. It uses the output feedback for controlling vibrations in flexible structures and is based on finding the closed-loop eigenstructures such that their eigenvectors are almost orthogonal to the open-loop eigenvectors. Most of the known eigenstructure assignment methods require a predetermination of the eigenstructure or at least the eigenvectors. A prior knowledge of the desired closed-loop system behavior in terms of the elements of its eigenvectors is not an easy task and is challenging in practice due to the fact that there are no one-to-one relationships between the elements of the eigenvectors and the states of the system. Predicting a desirable shape for the eigenvectors of a complicated system is not a straightforward process. In particular, for continuous systems, increasing the model's degrees of freedom makes the task of defining the desirable shape for eigenvectors even harder. Moreover, orthogonal eigenstructure control finds the closed-loop system eigenvectors within the achievable eigenvector sets that are orthogonal to the open-loop eigenvectors. As a result, the problem of algorithmic error due to the difference between achievable and desirable eigenvectors, which is common in many eigenstructure assignments methods, is eliminated. In this method, the eigenvectors of the closed-loop system are achievable eigenvectors that are consistent with the closed-loop eigenvalues; therefore, the excessive actuator forces observed in the earlier studies are eliminated. Because the eigenvectors or eigenvalues do not need to be predefined, the method can readily be applied to high-order systems.

This paper is structured as follows. In Sec. 2, the mathematics of orthogonal eigenstructure control is explained. Application of OEC to vibration cancellation of a plate is described in Sec. 2 and the effects of different operating eigenvalues are shown and discussed. Finally, the conclusion is given in Sec. 4.

2 Orthogonal Eigenstructure Control

A second order system of equations of the form

$$M\ddot{q} + D_d\dot{q} + K_s q = F_i u_i + F_d u_d \tag{1}$$

is considered where the mass, damping, and stiffness matrices are denoted by $n \times n$ matrices of M, D_d, and K_s, respectively. The control input matrix is F_i, the disturbance input matrix is F_d, and the displacement, velocity, and acceleration are presented by q, $\dot{q}$, and $\ddot{q}$, respectively. u_i is the external control vector and u_d is the external disturbance vector. The equation of motion (Eq. (1)) can be written in state-space form as

$$\dot{x} = Ax + Bu + Ef \tag{2}$$

where A is the $2n \times 2n$ state matrix, B is a $2n \times m$ input matrix, E is a $2n \times 1$ disturbance input matrix, f is the disturbance vector, and u is the input vector of dimension m.

The vector $x = \{q \;\; \dot{q}\}^T$ is the $2n \times 1$ state vector and $\dot{x}$ is the time derivative of the state vector. The output equation for the system can be written as

$$y = Cx \tag{3}$$

where y is the $m \times 1$ output vector and C is the $m \times 2n$ output matrix. Defining $m \times m$ feedback gain matrix K, the input control force and the closed-loop equation of motion are

$$u = Ky \tag{4}$$

$$\dot{x} = (A + BKC)x + Ef \tag{5}$$

The goal is to find the appropriate control gain K. For the closed-loop system of Eq. (5), the eigenvalue problem is defined as

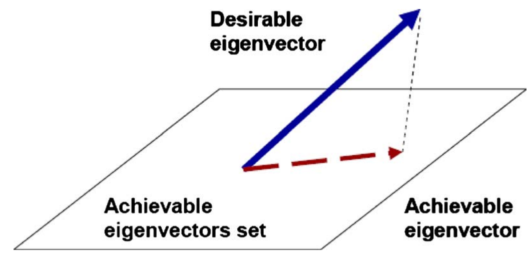

Fig. 1 The difference between an achievable and a desirable eigenvector

$$(A + BKC)\phi_i = \lambda_i \phi_i, \quad i = 1,2, \ldots ,2n \tag{6}$$

where ϕ_i and λ_i are the closed-loop eigenvectors and eigenvalues of the system, respectively. Equation (6) in matrix form is

$$[A - \lambda_i I | B]\begin{Bmatrix} \phi_i \\ KC\phi_i \end{Bmatrix}, \quad i = 1,2, \ldots ,2n \tag{7}$$

where I is the $2n \times 2n$ identity matrix. It can be seen that the vector $\left\{ \begin{smallmatrix} \phi_i \\ KC\phi_i \end{smallmatrix} \right\}$ is in the null space of the matrix $S_{\lambda i} = [A - \lambda_i I | B]_{2n \times (2n+m)}$. The null space of this matrix can also be found by its singular value decomposition. The two definitions of the null space of $S_{\lambda i}$ are used for finding the control gain matrix K. Calculating the singular value decomposition of $S_{\lambda i}$, one can write

$$S_{\lambda i} = [U_i]_{2n \times 2n}[\Sigma_i | 0_{2n \times m}]_{2n \times (2n+m)}[V_i^*]_{(2n+m) \times (2n+m)} \tag{8}$$

In OEC, λ_i is defined as the operating eigenvalues. We will show that if λ_i are chosen from the open-loop eigenvalue set, we are able to regenerate the open-loop system and simultaneously generate systems that have eigenvectors almost orthogonal to the eigenvectors of the regenerated open-loop system. The index i is used to specify the equation for the ith operating eigenvalue. The number of operating eigenvalues is the same as the number of the required pairs of actuators and sensors m. This study shows, through a set of examples, that the m farthest open-loop eigenvalues from the origin should be chosen as the operating eigenvalues. U_i and V_i are the left and right orthonormal matrices, respectively, and V_i^* is the conjugate transpose of the complex matrix V_i. If V_i is partitioned, then the second column block of V_i spans the null space of the $S_{\lambda i}$ [13,16],

$$[V_i]_{(2n+m) \times (2n+m)} = \begin{bmatrix} [V_{11}^i]_{2n \times 2n} & [V_{12}^i]_{2n \times m} \\ [V_{21}^i]_{m \times 2n} & [V_{22}^i]_{m \times m} \end{bmatrix} \tag{9}$$

Any linear combination of m columns of V_{12}^i is an achievable eigenvector of the closed-loop system; for a coefficient vector r^i, it implies that

$$\phi_i^a = V_{12}^i r^i \tag{10}$$

The corresponding control gain matrix K can be found using

$$KC\phi_i^a = V_{22}^i r^i \tag{11}$$

Most of the eigenstructure assignment methods define a desired eigenvector for the system ϕ_i^d using different approaches such as eigenvector shaping that uses a pseudo-inverse of V_{12}^i to find the required r^i [17]. Those methods have some limitations because the controlled eigenvectors will not be identical to the desired ones. In general, there is always a distance between the desired and controlled eigenvectors [17], as shown in Fig. 1.

The orthogonal eigenstructure control regenerates the open-loop system by finding the open-loop eigenvectors and their orthogonal vectors for the operating modes. In other words, the open-loop eigenvectors of the operating modes are defined by the intersections of the open-loop eigenvector set and the achievable eigenvector sets, as illustrated in Fig. 2.

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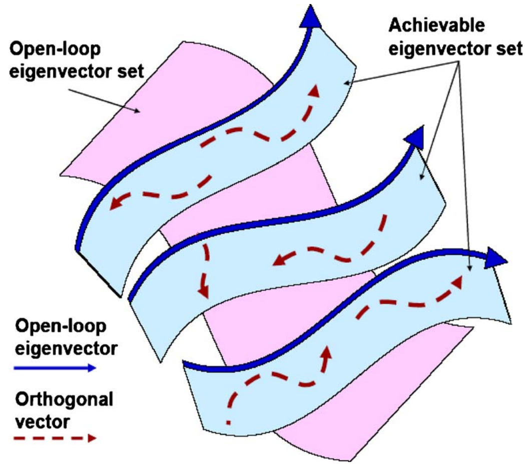

Fig. 2 Open-loop eigenvectors are the intersections of the open loop and achievable eigenvectors sets

To explain the orthogonal eigenstructure control, we define the modal energy corresponding to the ith eigenvector of the closed-loop system, using Eq. (10),

$$E_i = r^{i*}V_{12}^{i*}V_{12}^{i}r^{i} \tag{12}$$

Since V_{12}^{i} is complex, the $V_{12}^{i*}V_{12}^{i}$ is a Hermitian matrix, and its eigenvalue decomposition is

$$[V_{12}^{i*}]_{m\times 2n}[V_{12}^{i}]_{2n\times m} = \bar{U}^{i}\Lambda^{i}\bar{U}^{i*} \tag{13}$$

where $\bar{\Lambda}_i$ and $\bar{U}^i$ are the eigenvalue and eigenvector matrices of $V_{12}^{i*}V_{12}^{i}$. Similarly, for V_{22}^{i}, the eigenvalue decomposition of the Hermitian matrix $V_{22}^{i*}V_{22}^{i}$ is

$$[V_{22}^{i*}]_{m\times m}[V_{22}^{i}]_{m\times m} = \bar{U}_w^{i}\bar{\Lambda}_w^{i}\bar{U}_w^{i*} \tag{14}$$

where $\bar{\Lambda}_w^i$ and $\bar{U}_w^i$ are the eigenvalue and eigenvector matrices of $V_{22}^{i*}V_{22}^{i}$. It has been shown by the authors that the eigenvalues of the Hermitian products $V_{12}^{i*}V_{12}^{i}$ and $V_{22}^{i*}V_{22}^{i}$ belong to the [0 1] interval [13,14]. It has also been shown that the eigenvectors of $V_{22}^{i*}V_{22}^{i}$ and $V_{12}^{i*}V_{12}^{i}$ are identical, and the summation of the eigenvalues of $V_{12}^{i*}V_{12}^{i}$ and $V_{22}^{i*}V_{22}^{i}$ associated with similar eigenvectors are unity,

$$\bar{\Lambda}_w^i + \bar{\Lambda}^i = I \tag{15}$$

$$\bar{U}^i = \bar{U}_w^i \tag{16}$$

If the eigenvector $\bar{U}_J^i$ associated with a unity eigenvalue of $V_{12}^{i*}V_{12}^{i}$ is considered as r^i in Eq. (12), its modal energy $E^i=1$. Rearranging Eq. (13) yields

$$\bar{U}^{i*}V_{12}^{i*}V_{12}^{i}\bar{U}^{i} = \bar{\Lambda}^i \tag{17}$$

If $\bar{U}_J^i$ is the eigenvector corresponding to the unity eigenvalue, then

$$\bar{U}_J^{i*}V_{12}^{i*}V_{12}^{i}\bar{U}_J^{i} = 1 \tag{18}$$

and

$$\bar{U}_J^{i*}V_{22}^{i*}V_{22}^{i}\bar{U}_J^{i} = 0 \tag{19}$$

Equations (18) and (19) yield

$$V_{22}^{i}\bar{U}_J^{i} = 0 \tag{20}$$

which results in

$$KC\phi_i^a = V_{22}^{i}r^{i} = V_{22}^{i}\bar{U}_J^{i} = 0 \tag{21}$$

or $K=0$, which implies that the control gain K is zero, and the open-loop system has been regenerated. The selected r^i generates the open-loop eigenvectors from the null space of the closed-loop eigenvectors associated with the operating eigenvalue λ_i. $V_{12}^{i}\bar{U}_J^{i}$ is identical to the eigenvector corresponding to the open-loop eigenvalue or the operating eigenvalue. The other eigenvectors associated with nonunity eigenvalues of $V_{12}^{i*}V_{12}^{i}$ are orthogonal to the eigenvector associated with the unity eigenvalue of $V_{12}^{i*}V_{12}^{i}$. Therefore, a set of closed-loop eigenvectors can be found, which is orthogonal to the open-loop eigenvectors. The modal energies associated with the closed-loop eigenvectors are equal to nonunity eigenvalues of $V_{12}^{i*}V_{12}^{i}$. Since the nonunity eigenvalues of $V_{12}^{i*}V_{12}^{i}$ are small, the modal energies of the modes associated with the operating eigenvalues become zero or negligible.

The feedback gain matrix K is determined as

$$K = W(CV)^{-1} \tag{22}$$

V and W are calculated by appending the calculated closed-loop eigenvectors for all the operating eigenvalues,

$$V = [V_{12}^{1}r^{1}, \ldots, V_{12}^{m}r^{m}] \tag{23}$$

$$W = [V_{22}^{1}r^{1}, \ldots, V_{22}^{m}r^{m}] \tag{24}$$

Using the real part of K, one can write the state matrix of the closed-loop system as

$$A_c = A + BKC \tag{25}$$

The number of the eigenvectors of $V_{12}^{i*}V_{12}^{i}$ that can be chosen as r^i is m, which is the same as the number of actuators and operating eigenvalues. It results in the possibility of m^m solutions for the closed-loop system. Among the eigenvalues of $V_{12}^{i*}V_{12}^{i}$, there is one unity eigenvalue and $m-1$ zero or negligible ones. If the eigenvectors associated with the unity eigenvalues are selected as the coefficient vectors r^i, the open-loop system is regenerated. Excluding the regenerated open-loop system, there are m^m-1 possible closed-loop systems.

Figure 2 shows the open-loop eigenvectors and achievable closed-loop eigenvectors of a system with three collocated actuators and sensors. For each open-loop eigenvector, two orthogonal eigenvectors can be found that lie within achievable eigenvectors set. If some of the eigenvectors have not been changed, a closed-loop system can still be found. As a result, there are $3^3-1=26$ closed-loop systems that result from the control method.

In the next section, it is shown how the selection of the operating eigenvalues (from among the possible open-loop eigenvalues) can significantly influence the closed-loop systems that result from the method. If the operating eigenvalues are close to the origin, the resulting closed-loop systems have different behaviors and might even be unstable. Conversely, if the operating eigenvalues are far from the origin—for instance, choosing the farthest eigenvalues from the origin—the gain matrices converge together, and all the resulting closed-loop systems are nearly identical, thus eliminating the trial and error and the guesswork by the designer that is needed in other methods.

3 Vibration Control of a Plate

As an example of a continuous system, we consider a plate that is simply supported at four edges and apply the orthogonal eigenstructure control to suppress the vibration induced by the disturbance. The plate is assumed to be a square steel plate with length and width of 40 cm and a thickness of 1 mm. The Young's modulus of the material is 2.09×10^9 N/m^2, and the Poisson's ratio is 0.31. The finite element method has been used for modeling the plate. A code is written in MATLAB to simulate the response of the plate. Figure 3 shows the plate, the assigned nodes, and the elements on the plate.

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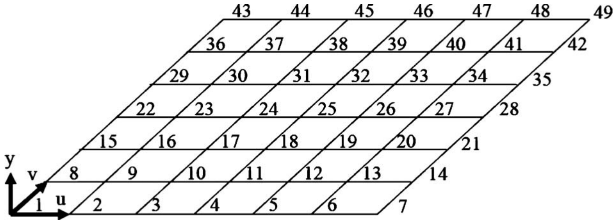

Fig. 3 Elements and nodes of the square plate. The plate is simply supported at all edges.

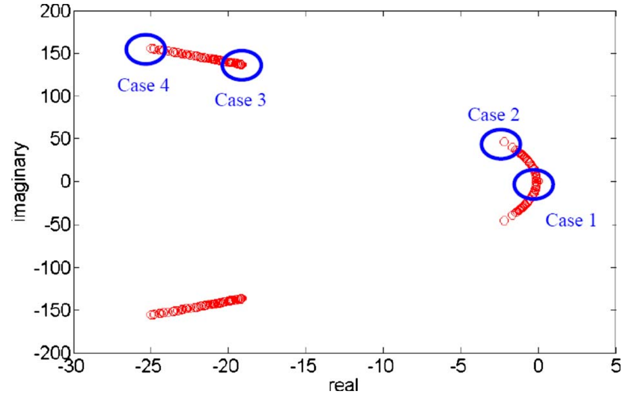

Fig. 4 Open-loop eigenvalues of the first order realization of the plate and four different areas for the operating eigenvalues

The Mindlin plate theory that includes transverse shear deformation has been used for defining the displacement field of the plate. The procedures for determining the local mass and stiffness matrices are reported in Ref. [18]. Linear quadrilateral elements have been used to model the plate. Each node has three degrees of freedom, including two in-plane displacements in u and v directions and the transverse displacement y. Since the model consists of 49 nodes, global mass and stiffness matrices, M and K_s in Eq. (1), are 149×149. For the damping matrix D_d, a linear damping $D_d=0.2M+0.002K_s$ after scaling the mass and stiffness matrices is assumed. When the second order system of Eq. (1) is transferred to a first order realization of Eq. (2), the dimension of the state matrix A is 298×298. The boundary conditions are applied, and the eigenvalues of the open-loop system is calculated and shown in Fig. 4. Four areas have been considered to choose the operating eigenvalues. The operating eigenvalues must be substituted in Eq. (8) to find the appropriate control gain matrix. Since using complex conjugates of the operating eigenvalues results in similar closed-loop systems, we use only the operating eigenvalues with positive imaginary parts. The disturbance force applied to the plate is a sine wave with a frequency of 2 kHz and an amplitude of 10 N. The disturbance force is normal to the plate at node 27 and causes bending in the plate. Three control actuators are considered to be on nodes 18, 25, and 32 in order to have the vibration cancellation, while they are not surrounding the disturbance source. The displacements of the same nodes are used for feedback, which indicates collocation of actuators and sensors. As stated earlier, there are 26 resulting closed-loop systems as the outcomes of the orthogonal eigenstructure control, and the most desirable closed-loop system has to be identified from them.

For case 1, the operating eigenvalues are $-0.2349+11.6115i$, $-0.1555+7.4495i$, and $-0.1301+5.4826i$. The possible closed-loop systems include 6 unstable and 21 stable systems. However, the only acceptable closed-loop system can be achieved when $r^1=\bar{U}_1^1$, $r^2=\bar{U}_2^2$, and $r^3=\bar{U}_1^3$. $r^i=\bar{U}_j^i$ implies that the coefficient vector r^i associated with the ith operating eigenvalue is the jth eigenvector in the eigenvector matrix $\bar{U}^i$ calculated using Eq. (13). For the rest of the stable systems, the control effort does not attenuate the vibration. The real part of the control gain matrix, which yields a desirable closed-loop system, is

$$K=\begin{bmatrix} -540.8461 & -374.6068 & 211.4074 \\ -700.4144 & 196.5303 & -168.8743 \\ -560.3060 & -382.0424 & 242.0038 \end{bmatrix}$$

Figure 5 shows the closed-loop and open-loop eigenvalues of the systems for cases 1–4. Figure 5(*a*) shows the eigenvalues of the open-loop and closed-loop systems for case 1. It can be noted that the closed-loop poles have not displaced out of the open-loop pole locus and remain in the vicinity of the open-loop poles. Also, actuation forces for cases 1–4 are shown in Fig. 6. Figure 6(*a*) shows the actuation forces for case 1. The maximum actuation forces are 11.2 N on nodes 18, 15.8 N on node 25, and 11.6 on node 32. It is seen that the actuation forces are larger than the disturbance force magnitude. Figure 7 shows the displacements of the nodes of the plate for open-loop and closed-loop systems in case 1. As it can be seen, the attenuation of vibration is not significant (Fig. 6).

In case 2, the operating eigenvalues are $-1.8767+42.1091i$, $-1.4634+36.8949i$, and $-1.2746+34.2488i$. The possible closed-loop systems include 12 unstable and 15 stable systems. Only three of the stable systems are unacceptable since their control efforts do not suppress the vibration. The most desirable closed-loop system, which has the acceptable vibration suppression and

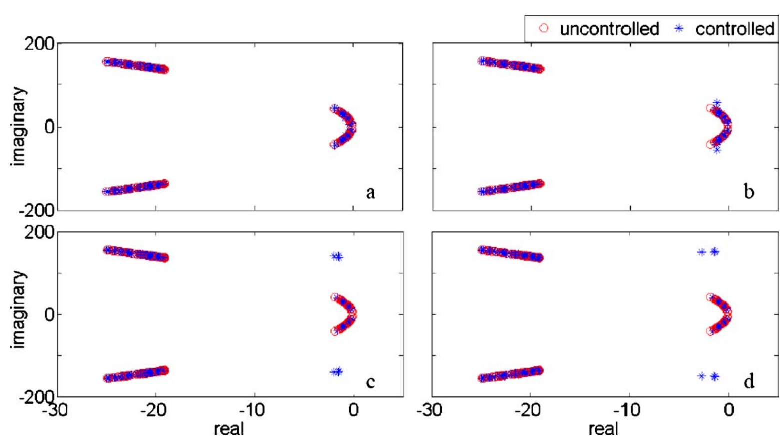

Fig. 5 Eigenvalues of the open-loop and closed-loop systems for different cases: (*a*) case 1, (*b*) case 2, (*c*) case 3, and (*d*) case 4

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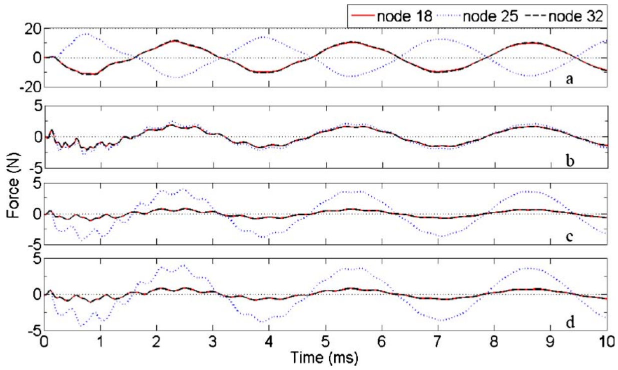

Fig. 6 Actuation forces for different cases: (*a*) case 1, (*b*) case 2, (*c*) case 3, and (*d*) case 4

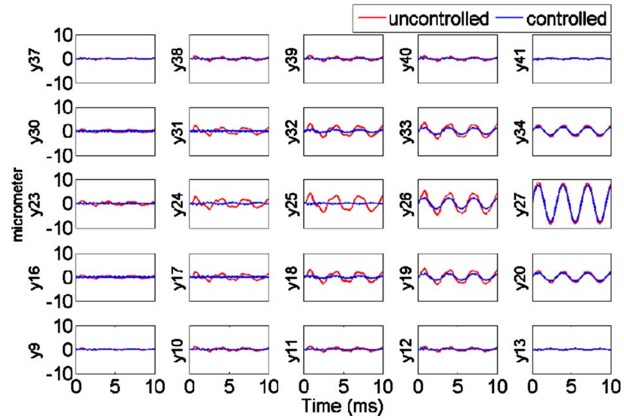

Fig. 8 Transverse displacement at nodes of the plate for case 2

low actuation forces, can be reached when $r^1=\bar{U}_1^1$, $r^2=\bar{U}_2^2$, and $r^3=\bar{U}_1^3$. The real part of the control gain matrix is

$$K = 1.0 \times 10^3 \begin{bmatrix} -0.9266 & -0.9672 & -1.7274 \\ -0.0556 & -0.5020 & 0.6328 \\ -0.7657 & -0.9279 & -1.7156 \end{bmatrix}$$

Figure 8 shows the displacement of the nodes of the plate. A good vibration cancellation can be seen, especially at the nodes that are not between the source of disturbance and the control actuators. The distributions of the poles of the closed-loop and open-loop systems are depicted in Fig. 5(*b*). It is seen that two pairs of the closed-loop poles are moved away from the locus of the open-loop poles. Figure 6(*b*) shows the actuation forces at different nodes. The maximum actuation forces are 1.86 N on node 18, 2.41 N on node 25, and 1.84 on node 32. The actuation forces are smaller than the disturbance force and considerably smaller than those of case 1. Better vibration suppression in comparison to case 1 is achieved.

In case 3, the operating eigenvalues are −19.1368+136.6404i, −19.1094+136.5437i, and −19.1094+136.5437i. Figure 9 shows the displacement of different nodes of the plate and the suppression of vibration. The control efforts significantly reduce the vibration in the plate. All the 26 control gain matrices of the resulting closed-loop systems converge to

$$K = 1.0 \times 10^4 \begin{bmatrix} -1.8568 & -0.0263 & 0.0465 \\ -0.0532 & -1.8832 & -0.0536 \\ 0.0444 & -0.0271 & -1.8600 \end{bmatrix}$$

It is interesting to see that the off-diagonal elements of the gain matrix are two orders of magnitude smaller than the diagonal elements. This shows that the control is decoupled. In this case, three pairs of the closed-loop poles are moved away from the open-loop pole cluster, as shown in Fig. 5(*c*). The distance between the displaced poles in case 3 is larger than the distance of the displaced poles of case 2. Figure 6(*c*) shows the actuation forces. The maximum actuation forces are 0.85 N on node 18, 3.92 N on node 25, and 0.85 on node 32. A comparison between the actuation forces of cases 3 and 2 shows that in case 3, the fluctuation of the amplitudes of the control forces is slower than that in case 2.

For case 4, the operating eigenvalues are −24.9799+155.743i, −24.9799+155.743i, and −24.8108+155.2264i. The closed-loop systems are identical, and the control gain matrices converge to

$$K = 1.0 \times 10^4 \begin{bmatrix} -2.1998 & 0.0196 & 0.0156 \\ 0.0127 & -2.1497 & 0.0379 \\ -0.0137 & -0.0719 & -2.2532 \end{bmatrix}$$

The control gain matrix in this case, similar to case 3, has small off-diagonal elements in comparison to the diagonal elements. The suppression of vibration in the nodes of the plate can be seen in Fig. 10. Figure 5(*d*) shows the distributions of the open-loop and closed-loop poles. It shows that three pairs of the closed-loop

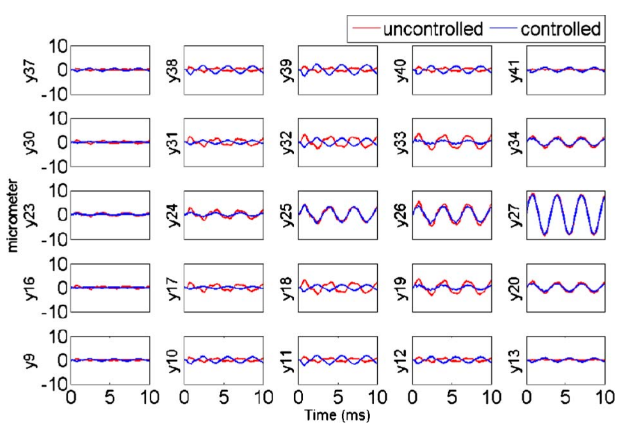

Fig. 7 Transverse displacement at nodes of the plate for case 1

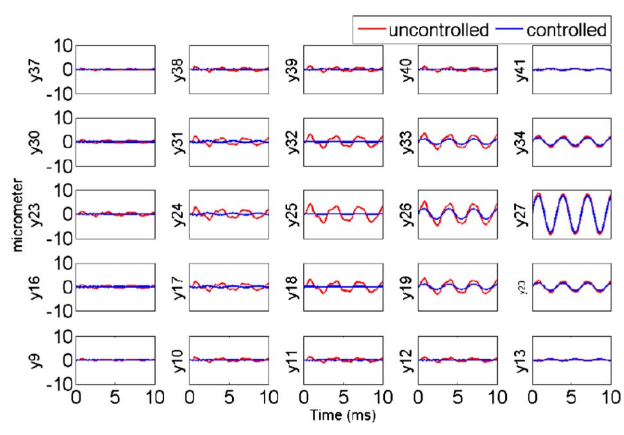

Fig. 9 Transverse displacement at nodes of the plate for case 3

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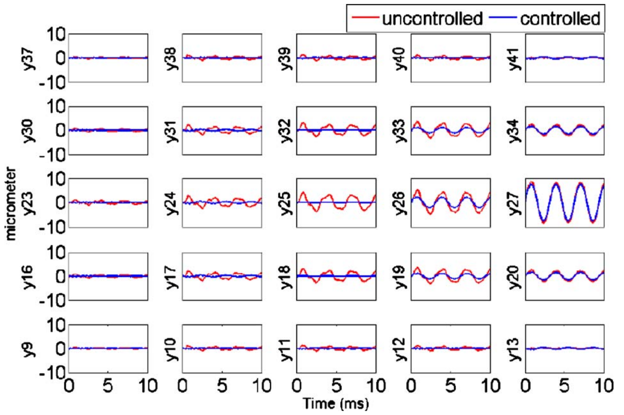

Fig. 10 Transverse displacement at nodes of the plate for case 4

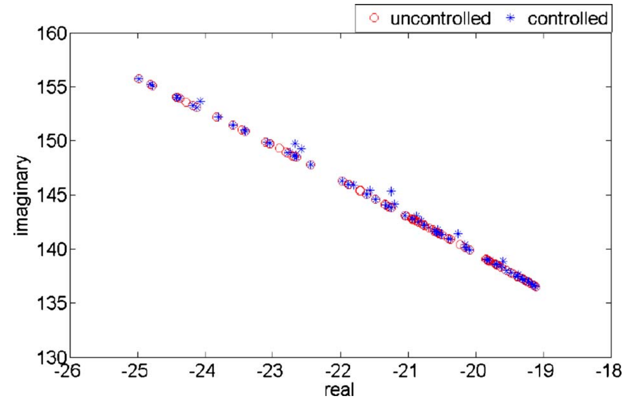

Fig. 12 Distribution of eigenvalues in cluster far from origin at case 4

poles moved away from the locus of the open-loop poles. They moved slightly further from the poles in case 3. Actuation forces can be seen in Fig. 6(*d*). The maximum actuation forces are 0.85 N on node 18, 3.94 N on node 25, and 0.85 on node 32. Interestingly, the actuation forces in case 4 are almost identical to those in case 3.

To examine the effect of larger diagonal elements in comparison to small off-diagonal elements of the control gain matrix in cases 3 and 4, we have set the off-diagonal elements equal to zero and have simulated the closed-loop systems again. The displacement plots have not altered noticeably. The same similarity can be seen in the plots of actuation forces when the off-diagonal elements are set to zero, and the results are compared with the original results. Therefore, it can be used to decouple the feedback signals of each actuator from the other feedback signals if the operating eigenvalues are chosen appropriately.

A closer look at the closed-loop poles in case 4 shows that almost all the eigenvalues have slightly moved. Figure 11 shows the right cluster poles with positive imaginary parts. It is not possible to relate any of the closed-loop poles to the open-loop ones. With an exception of three of the eigenvalues that moved significantly, the rest of them have moved within the open-loop pole locus. Figure 12 depicts the left cluster poles with positive imaginary parts. Eight of the poles have moved slightly away from the open-loop pole locus, and the rest of them have moved within the locus. Therefore, low actuation forces are needed to move the poles away from the open-loop locations, as stated by Slater and Zhang [11].

To further investigate the effect of the operating eigenvalues, complex numbers with real parts within [−30−2] and imaginary parts within [0 200] have been used as operating eigenvalues. The increment in the imaginary parts is 5, and that in the real parts is 2. Since three operating eigenvalues are needed, if the middle one is chosen to be −20+175i, for example, the two other operating eigenvalues are −20+174i and −20+176i. Figure 13 shows the operating eigenvalues that result in desirable closed-loop systems that attenuate the vibration significantly. If they are chosen from the left part, the control gain matrices for all the possible closed-loop systems are converged to one matrix, in which the diagonal elements are larger than the off-diagonal ones. Also, less fluctuation in their actuation forces can be seen. Further investigations are needed to find any possible relation between the model parameters of different systems and the distribution of the operating eigenvalues that may result in a robust control. The practical procedure of application of orthogonal eigenstructure control is suggested as follows:

1. Define state-space realization of the system. Calculate the eigenvalues of the system. Determine the m greatest eigenvalues of the open-loop system λ_i, where m is the number of the pairs of actuators and sensors.
2. Using the ith λ_i, define the nonsquare matrix $S_{\lambda i}=[A-\lambda_i I|B]$. Calculate the singular value decomposition of $S_{\lambda i}$. Partition the right unitary matrix V_i and define V_{12}^i and V_{22}^i, as described in Eq. (9).

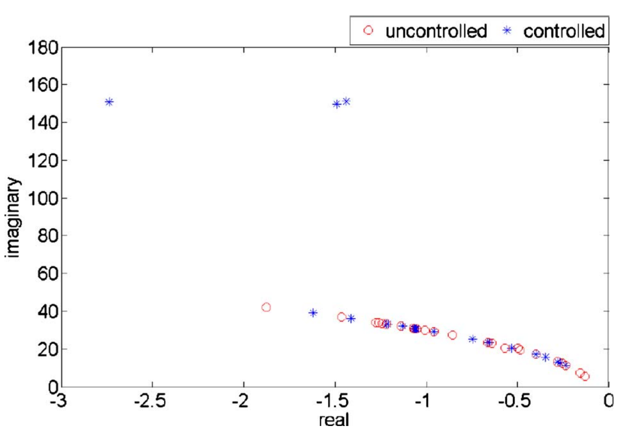

Fig. 11 Distribution of eigenvalues in cluster close to origin at case 4

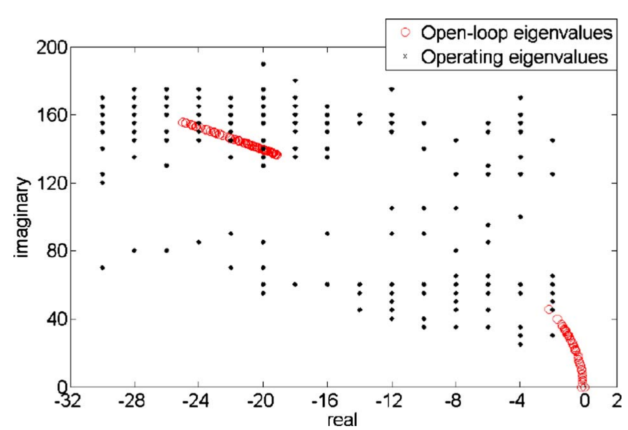

Fig. 13 Distribution of operating eigenvalues for case 4 that generate acceptable closed-loop systems

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3. Calculate $V_{12}^{i*}V_{12}^{i}$ from the previous step. Find the eigenvalue matrix $\bar{\Lambda}^i$ and the eigenvector matrix $\bar{U}^i$ for the Hermitian matrix $V_{12}^{i*}V_{12}^{i}$ using Eq. (13).
4. Pick the jth eigenvector $\bar{U}_j^i$ associated with the jth eigenvalue λ_j^i. Define $r^i=\bar{U}_j^i$. Calculate $v^i=V_{12}^i r^i$ and $w^i=V_{22}^i r^i$.
5. Repeat steps 2–4 m times and find v^i and w^i for all the operating eigenvalues. Define matrices V and W, as described in Eqs. (23) and (24).
6. Find the gain matrix K using Eq. (22).

4 Conclusion

The finite element model of a steel plate has been used to show the application of the orthogonal eigenstructure control for vibration cancellation. A sinusoidal disturbance has been applied to the plate, and three actuators have been used to reduce the vibration in the plate. The effect of the operating eigenvalues has been shown by choosing them from different areas of the locus of the open-loop eigenvalues. When the closest open-loop eigenvalues to origin are chosen as operating eigenvalues, the resulting set of closed-loop systems contains only one stable closed-loop system. By choosing the open-loop eigenvalues farther from the origin, we showed that the resulting closed-loop systems converge due to the convergence of their respective control gain matrices. The control scheme has decoupled channels since gain matrices have diagonal elements several orders of magnitude larger than the off-diagonal elements. Also, the actuation forces become significantly smaller than the disturbance. To further investigate the operating eigenvalues and to find the map of appropriate operating eigenvalues, a range of complex numbers are used as operating eigenvalues. It is shown that the open-loop eigenvalues are confident candidates for the operating eigenvalues; however, the operating eigenvalues are not limited to the open-loop eigenvalue set.

Nomenclature

A = open-loop state matrix
A_c = closed-loop state matrix
B = input matrix
C = output matrix
D_d = damping matrix
E = disturbance input matrix
E_i = modal energy of the ith mode
f = disturbance
I = identity matrix
K = gain matrix
K_s = stiffness matrix
m = number of inputs (actuators/sensors)
M = mass matrix
n = dimension of the second order system
r^i = vector of coefficient
$S_{\lambda i}$ = augmented matrix associated with λ_i
U_i = left unitary matrix of $S_{\lambda i}$
$\bar{U}^i$ = eigenvalue matrix of $V_{12}^{i*}V_{12}^{i}$ and $V_{22}^{i*}V_{22}^{i}$
$\bar{U}_w^i$ = eigenvalue matrix of $V_{22}^{i*}V_{22}^{i}$, equal to $\bar{U}^i$
$\bar{U}_j^i$ = eigenvalue of $V_{12}^{i*}V_{12}^{i}$ associated with nonunity eigenvalues
$\bar{U}_J^i$ = eigenvalue of $V_{12}^{i*}V_{12}^{i}$ associated with unity eigenvalue
V_i = right unitary matrix of $S_{\lambda i}$
V_{12}^i = upper part of N^i
V_{22}^i = lower part of N^i
V = appended matrix of $V_{12}^i r^i$
W = appended matrix of $V_{22}^i r^i$
x = state vector
$\dot{x}$ = time derivative of state vector
ϕ_i = ith closed-loop eigenvalue
ϕ_i^a = achievable eigenvector of the ith mode
λ_i = ith operating eigenvalue
$\bar{\lambda}_j^i$ = eigenvalues of $V_{12}^{i*}V_{12}^{i}$
$\bar{\Lambda}_i$ = eigenvalue matrix of $V_{12}^{i*}V_{12}^{i}$
$\bar{\Lambda}_w^i$ = eigenvalue matrix of $V_{22}^{i*}V_{22}^{i}$
N^i = matrix that spans the null space of the ith mode
Σ_i = matrix of singular values of $S_{\lambda i}$

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