Actuators' Locations in Vibration Cancellation of a Plate Using Orthogonal Eigenstructure Control

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Abstract

Orthogonal eigenstructure control is a novel control method that recently has been developed by the authors as a method for active vibration cancellation. This method is a feedback control method, which is applicable to linear multi-input multi-output systems. The remarkable advantage of this method over most of the eigenstructure assignment methods is that it neither needs pole placement nor eigenvector shaping. By eliminating the requirement for defining desirable eigenvectors, the algorithmic error due to difference between achievable and desirable eigenvectors is no longer exist. Moreover, since the closed-loop poles are not required to be placed, the possibilities for excessive actuation forces are eliminated. The orthogonal eigenstructure control regenerates the open-loop eigenvectors and simultaneously finds the vectors orthogonal to them. Replacing the open-loop eigenvectors with those orthogonal vectors results in vibration suppression in the system. Furthermore, by eliminating the concept of desirable eigenvector for the closed-loop system, the method has become suitable for the system with high degrees of freedom. To show that this control method is easily applicable to system with high degrees of freedom, it is applied to a finite element model of a plate, and the effect of the location of the control actuators in the cancellation of vibration has been investigated.

Introduction

Eigenstructure assignment in linear multi-input multi-output systems was first introduced by Moore. He showed that the control gains, or in other words eigenvectors, for placing the eigenvalues in pre-defined locations are not unique. In fact, there is a class of eigenvectors associated with a distinct set of closed-loop eigenvalues [1, 2]. Then, the idea of eigenstructure assignment has initiated which is both placement of eigenvalues in desired locations and choosing or shaping a set of proper eigenvectors. Obviously, the speed of the response is related to the assigned closed-loop eigenvalues, and the shape of the response can be adjusted by the assigned eigenvectors. Cunningham [3] used singular value decomposition for defining the class of achievable eigenvectors. Using singular value decomposition, one can define the vectors that span the null space of the closed-loop eigenvectors. He applied this technique to an output feedback control by optimally combining the basis vectors in order to minimize the error between achievable and desirable eigenvectors.

Shelly et al. proposed eigenvector scaling which was a mode localization technique. This method alternate specific elements of each closed-loop eigenvector in order to uniformly decrease the relative displacement of specific areas in the system [4]. They showed analytically that attenuation is independent of the type of the disturbance. Further, they introduced an active control method called eigenstructure shaping. This method modifies part or the entire system mode shapes and refines the behavior of the system [5]. A Moore-Penrose generalized left inverse has been used, since it gives the minimum Euclidean 2-norm error, to produces the closest eigenvector in least square sense to the desired ones. Tang et al. [6, 7] proposed optimal eigenvectors calculation using Rayleigh principle by minimizing the ratio of modal energy at the concerned area to the modal energy of the whole structure using an auxiliary eigenvalue problem. They showed that pre-defining the desired eigenvector components can cause unsatisfactory performance if a match between components of the desired and achievable eigenvectors happens in the unimportant degrees of freedom [8].

Orthogonal eigenstructure control, for feedback control of linear multi-input multi-output systems, has been recently developed by the authors [9-12] to address some existing problems in eigenstructure assignment methods. One of the significant drawbacks of some of the existing methods is that the desirable eigenvectors that are required to be defined by the control designer may not lie within the achievable eigenvectors set as shown on Figure 1. This leads to an algorithmic error, unsatisfactory behavior, or excessive actuation forces in the closed-loop system.

The orthogonal eigenstructure control does not need any pre-defining or shaping the closed-loop eigenvectors. Also, there is no need to place the closed-loop poles. The control designer just needs the open-loop system model, and its natural frequencies.

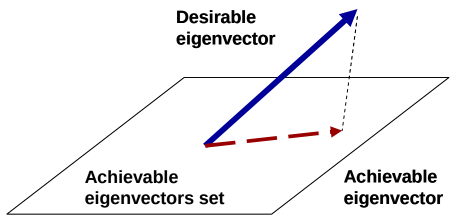

Figure 1. The difference between achievable and desirable eigenvectors

Depend on the number of the actuators that are needed, open-loop eigenvalues are used to define separate sets that span the null space of the closed-loop system. Each set is able to define the open-loop eigenvector associated with the open-loop eigenvalue that is used. Moreover, the algorithm finds more vectors that are orthogonal to that specific open-loop eigenvector while they lie within the achievable eigenvector set and substitute the open-loop eigenvectors with their orthogonal vectors. Therefore, the errors due to the difference between the achievable and desirable eigenvectors are eliminated, since the closed-loop eigenvectors are already within the achievable eigenvector set. The desirable eigenvectors are no longer required to be defined as illustrated on Figure 2. If there have been used m actuators, there are m^m possible closed-loop systems. Excluding the regenerated open-loop system, which is basically associated with zero gain matrix, there are m^m -1 closed-loop systems as the outputs of the control method. The designer chooses the most desirable system by evaluating their responses. In this method, the selected open-loop eigenvalues are called operating eigenvalues and they should be the greatest open-loop eigenvalues, in order to reach to a desirable closed-loop system. When the operating eigenvalues are far from the origin, all the possible closed-loop systems merge to one system. In this case, the diagonal elements of the control gain matrix are several order of magnitudes larger than the off-diagonal elements, which implies the control have resulted in the decoupled modes of closed-loop system.

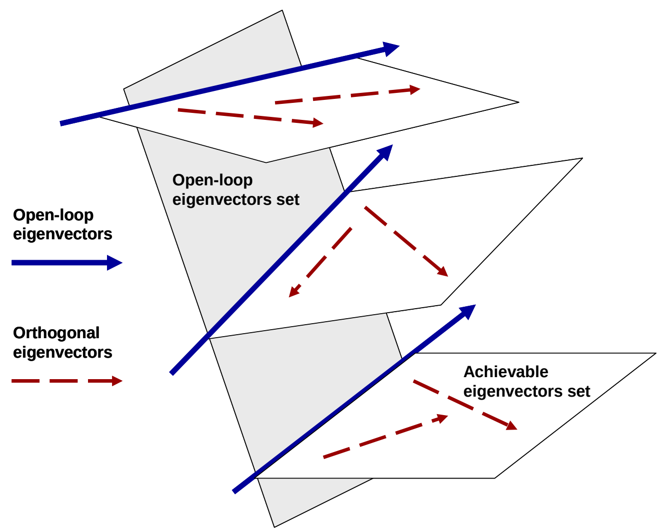

Figure 2. Open-loop eigenvectors are the intersections of the open-loop and achievable eigenvectors sets.

Since, there is no one-to-one relationship between the elements of the eigenvectors and the states of the system, defining eigenvectors for the closed-loop system becomes a very hard task. Predicting a desirable shape for the eigenvectors of a complicated system is not a straightforward procedure. Moreover, increasing the model's degrees of freedom makes the task of defining the desirable shape for eigenvectors even harder, such as the models of continuous systems. In this method, the eigenvectors of the closed-loop system are achievable eigenvectors and the closed-loop eigenvalues are consistent with them, so, the excessive actuator forces are prevented as stated by Slater et al. [13]. Since no definitions for the eigenvectors or eigenvalues are needed, this method is greatly applicable to the systems with high degrees of freedom. As an example of such systems; we apply the method for vibration cancellation in a plate. The plate has been modeled using finite element method. Further, the effect of the location of the control actuators with respect to the disturbance has been investigated. In the next section, we discuss the mathematical basis of the method.

Orthogonal Eigenstructure Control

Consider the first order realization of a closed-loop multi-input multi-output linear system

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\} + [E]\{f\} \tag{1}$$

$$\{y\} = [C]\{x\} \tag{2}$$

$$\{u\} = [K]\{y\} \tag{3}$$

$\{x\}$ is the $2n \times 1$ state vector, $[B]$ is $2n \times m$ input matrix, where $m \geq 2$ is the number of the actuators, $\{f\}$ is the disturbance vector with appropriate dimensions. $\{u\}$ is the input vector of dimension m. We assume the actuators and sensors are collocated; therefore, the output vector $\{y\}$ is $m \times 1$, and the output matrix $[C]$ is $m \times 2n$. $[K]$ is $m \times m$ feedback gain matrix. The combined closed-loop equation of motion is

$$\{\dot{x}\} = [A + BKC]\{x\} + [E]\{f\} \tag{4}$$

Defining ϕ_i as the closed-loop eigenvectors, λ_i as the operating eigenvalues, and I the $2n \times 2n$ identity matrix, one can define the eigenvalue problem as follow

$$[A - \lambda_i I \quad | \, B]\begin{Bmatrix} \phi_i \\ KC\phi_i \end{Bmatrix} = 0 \quad i = 1, \ldots, 2n \tag{5}$$

Vector $\begin{bmatrix} \phi_i^T & (KC\phi_i)^T \end{bmatrix}^T$ defines the basis of the null space of the matrix $S_{\lambda i} = [A - \lambda_i I \quad | \, B]_{2n \times (2n+m)}$, since their product is zero. Singular value decomposition of $S_{\lambda i}$ implies

$$S_{\lambda i} = [U_i]_{2n \times 2n}[\Sigma_i \,|\, 0_{2n \times m}]_{2n \times (2n+m)}[V_i^*]_{(2n+m) \times (2n+m)} \tag{6}$$

$[U_i]$ and $[V_i]$ are the left and right orthonormal matrices respectively, and $[V_i^*]$ is the conjugate transpose of the complex matrix $[V_i]$. Partitioning $[V_i]$

$$[V_i]_{(2n+m) \times (2n+m)} = \begin{bmatrix} [V_{11}^i]_{2n \times 2n} & [V_{12}^i]_{2n \times m} \\ [V_{21}^i]_{m \times 2n} & [V_{22}^i]_{m \times m} \end{bmatrix} \tag{7}$$

An achievable eigenvector ϕ_i^a of the closed-loop system is any linear combination of m columns of $[V_{12}^i]$ using an appropriate coefficient vector r^i.

$$\phi_i^a = [V_{12}^i]\{r^i\} \tag{8}$$

The control gain matrix $[K]$ is defined as follow

$$KC\phi_i^a = [V_{22}^i]\{r^i\} \tag{9}$$

To find the appropriate r^i, we use the modal energy corresponding to the i th achievable eigenvector of the closed-loop system

$$E_i = r^{i*}[V]^*[V_{12}^i]r^i \tag{10}$$

$[V_{12}^i]^*[V_{12}^i]$ and $[V_{22}^i]^*[V_{22}^i]$ are Hermitian matrices and their eigenvalue decompositions are

$$[V_{12}^i]^*_{N\times m}[V_{12}^i]_{N\times m} = \bar{U}^i \Lambda^i \bar{U}^{i*} \tag{11}$$

$$[V_{22}^i]^*_{m\times m}[V_{22}^i]_{m\times m} = \bar{U}_w^i(\bar{\Lambda}_w^i)\bar{U}_w^{i*} \tag{12}$$

$\bar{\Lambda}_i$ and $\bar{U}^i$ are the eigenvalues and eigenvectors matrices of $[V_{12}^i]^*[V_{12}^i]$, and $\bar{\Lambda}_w^i$ and $\bar{U}_w^i$ are the eigenvalue and eigenvector matrices of $[V_{22}^i]^*[V_{22}^i]$. It has been shown by the authors [10, 11] that

$$\bar{\Lambda}_w^i + \bar{\Lambda}^i = I \tag{13}$$

$$\bar{U}^i = \bar{U}_w^i \tag{14}$$

and the eigenvalues of the Hermitian products $[V_{12}^i]^*[V_{12}^i]$ and $[V_{22}^i]^*[V_{22}^i]$ belong to the interval $[0 \quad 1]$. Re-arranging the equation (11) implies

$$\bar{U}^{i*}[V_{12}^i]^*[V_{12}^i]\bar{U}^i = \bar{\Lambda}^i \tag{15}$$

If the eigenvector $\bar{U}_J^i$ associated with a unity eigenvalue of $[V_{12}^i]^*[V_{12}^i]$ in equation (11) is considered as r^i, its modal energy $E^i = 1$.

$$\bar{U}_J^{i*}[V_{12}^i]^*[V_{12}^i]\bar{U}_J^i = 1 \tag{16}$$

Using equation (13), we may have

$$\bar{U}_J^{i*}([V_{22}^i]^*[V_{22}^i])\bar{U}_J^i = 0 \tag{17}$$

equations (16) and (17) yield

$$[V_{22}^i]\bar{U}_J^i = 0 \tag{18}$$

That is the gain matrix is zero

$$KC\phi_i^a = [V_{22}^i]r^i = [V_{22}^i]\bar{U}_J^i = 0 \tag{19}$$

this implies that the open-loop system has been regenerated. We have seen that if the eigenvector $\bar{U}_J^i$ associated with a unity eigenvalue of $[V_{12}^i]^*[V_{12}^i]$ is selected as r^i, the open-loop eigenvectors within the null space of the closed-loop eigenvectors associated with the operating eigenvalue λ_i is regenerated. The open-loop eigenvectors are the intersections of the open-loop eigenvector set and the achievable eigenvector set. Other eigenvectors associated with non-unity eigenvalues of $[V_{12}^i]^*[V_{12}^i]$ are orthogonal to the eigenvector associated with the unity eigenvalue. Therefore, a set of closed-loop eigenvectors can be found that are orthogonal to the open-loop ones.

Appending the calculated closed-loop eigenvectors for all the operating eigenvalues

$$V = \left[[V_{12}^1]r^1 \cdots [V_{12}^m]r^m\right] \tag{20}$$

$$W = \left[[V_{22}^1]r^1 \cdots [V_{22}^m]r^m\right] \tag{21}$$

The control gain matrix K and the state matrix of the closed-loop system are

$$K = W(CV)^{-1} \tag{22}$$

$$A_c = A + BKC \tag{23}$$

Case Study

Figure 3 shows a plate that has been modeled using finite element method. The plate is made of steel with Young's modulus of 2.09×109 N/m2 and Poisson's ratio of 0.33. The plate is a square plate, with length of 40 cm, and thickness of 1 mm. The plate is simply supported at four edges. Mindlin plate theory is used for defining the displacement field of the plate. There are three degrees of freedom for each node, two degrees of freedom are in-plane displacements in u and v directions and third one is the transverse displacement y. Linear quadrilateral elements have used for finding mass and stiffness matrices of each element. There are 49 nodes on the plate; therefore, the dimensions of global mass and stiffness matrices are 149×149. Damping is assumed to be a linear. The dimension of the state matrix A of the first order realization of the system in equation (1) is 298×298. The disturbance force applied to the plate is a sine wave with 2 kHz frequency and amplitude of 10 N. The disturbance force is applied to the plate on node 27 and is normal to the plate which causes bending stresses in the plate. Two cases have been considered. In the first case the control actuators are on nodes 19, 26, and 33. In the second case, the control actuators are located on nodes 18, 25, and 32.

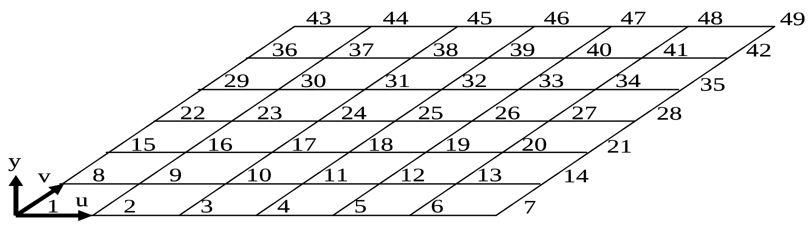

Figure 3. Simply supported plate with 49 nodes and 36 elements

Figures 4 and 5 are the time history of the displacements of the nodes of the plate for two cases. Great vibration isolation can be seen for the nodes with control actuators, and the ones beyond them. Vibration isolation for the nodes between the disturbance source and the control actuators, however, is not as significant as is at the nodes beyond the control actuators. Gain matrices for the case 1 and 2 are

$$K = 1.0\times 10^{4}\begin{bmatrix} -2.1656 & 0.0420 & 0.0267 \\ 0.0034 & -2.1985 & 0.0030 \\ -0.0319 & -0.0444 & -2.2220 \end{bmatrix} \quad \text{and} \quad K = 1.0\times 10^{4}\begin{bmatrix} -2.2371 & -0.0194 & -0.0079 \\ 0.1726 & -2.1124 & 0.0402 \\ -0.1508 & -0.0939 & -2.2536 \end{bmatrix}$$

respectively. The controls are decoupled, as it can be seen that the diagonal elements of the control gain matrices, which are two to three orders of magnitude greater than the off-diagonal elements.

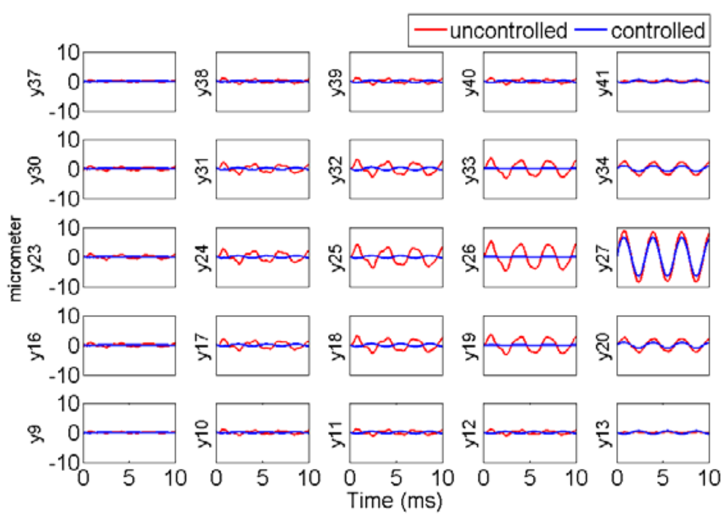

Figure 4. Case 1, actuators on nodes 19, 26, and 33

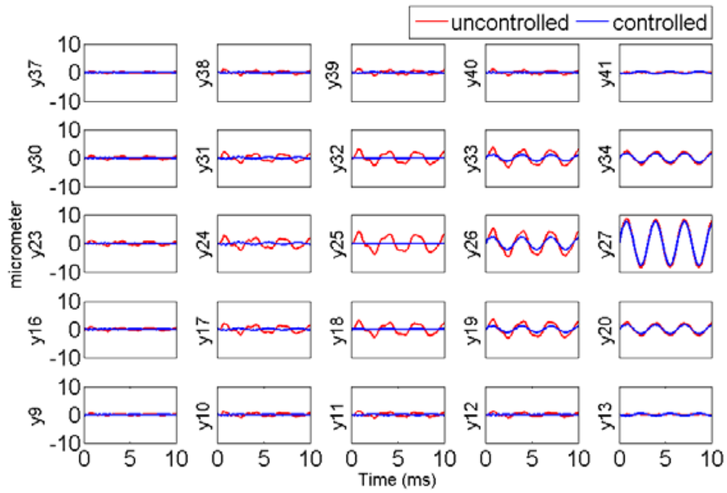

Figure 5. Case 2, actuators on nodes 18, 25, and 32

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