Application of orthogonal eigenstructure control to flight control design

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ABSTRACT

Orthogonal eigenstructure control is used for designing a control law that decouples the dynamic modes of a flying vehicle. Orthogonal eigenstructure control is a feedback control method for linear time invariant multi-input multi-output systems. This method has been recently developed by authors. The advantage of this control method over eigenstructure assignment methods is that there is no need for defining the closed-loop poles or shaping the closed-loop eigenvectors. This method eliminates the error due to the difference between achievable and desirable eigenvectors, by finding vectors orthogonal to the open-loop eigenvectors within the achievable eigenvectors set and replacing the open-loop eigenvectors with them. This method is also applicable to the systems with non-collocated actuators and sensors. Application of this method for designing a flight control law for the lateral directional dynamics of an F-18 HARV is presented, and compared to the results of an eigenstructure assignment method. In this case study, the actuators and sensors are not collocated. It is shown that the application of the orthogonal eigenstructure control results in a more significant dynamic modes decoupling in comparison to the application of the eigenstructure assignment technique.

Keywords: Orthogonal eigenstructure control, flight control design

1. INTRODUCTION

When a flying vehicle performs some maneuvers, a coupled response is generated, that may be undesirable from a performance point of view. Since vehicles can be modeled as multi-input multi-output systems, eigenstructure assignment methods have been used extensively for designing the flight control law. An eigenstructure assignment method has the ability to change the eigenvalues and eigenvectors of the systems in order to achieve better performances. Eigenstructure assignment methods, however, need the designer input for shaping the eigenvectors of the system. In general, there is no one-to-one relation between the states of the system and the elements of the eigenvectors of the closed-loop system, acknowledging that the response of a linear system is a linear combination of the eigenvectors. Moreover, there is always a difference between the achievable and desirable eigenvectors. This leads to an algorithmic error that may cause excessive actuation forces and undesirable behavior of the system.

Orthogonal eigenstructure control has recently been developed by the authors to address the aforementioned issues. This method has been primarily developed for vibration cancellation in structures. Even though this method has emerged form the eigenstructure assignment concept, but has substantial differences from them. This new method does not need defining the desirable eigenvectors for the closed-loop systems. Therefore, the error due to the difference between the desirable and achievable eigenvectors has been eliminated. Also, no pole placement is required; hence, the closed-loop poles are consistent with the closed-loop eigenvectors. Orthogonal eigenstructure control does not need any prediction regarding the closed-loop system characteristics and the only information that a controller designer needs is the model of the open-loop system. The method is straightforward and generates a class of possible closed-loop systems with eigenvectors orthogonal to the open-loop system eigenvectors. All the closed-loop eigenvectors lie within the achievable eigenvectors set.

The idea of eigenstructure assignment was initiated by Moore [1, 2], when he showed that there is a class of eigenvectors associated with a distinct set of closed-loop eigenvalues. Therefore, there are infinite number of control gains or eigenvector matrices for some given closed-loop eigenvalues. Cunningham [3] used singular value decomposition to find the vectors that span the null space of the eigenvectors of the closed-loop systems. Shelly et al. studied the vibration

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isolation problem in structures and showed analytically that absolute displacements in isolated areas can be reduced by eigenvector shaping, regardless of the type of the disturbance [2, 4, 5]. Later, they proposed SVD-eigenvector shaping using a Moore-Penrose generalized left inverse to produce the closest eigenvector in least square sense to the desired ones [6]. Slater et al. [7] showed if we only change the eigenvectors, the control efforts are not necessarily minimized when the closed-loop eigenvalues are forced to be close to the open-loop ones. They showed that a large change in eigenvectors may need a large movement of the eigenvalues to minimize the feedback gains. They also showed that closed-loop eigenvalues have to be consistent with eigenvectors to avoid large control efforts. Furthermore, since – at the time – there was no method to have closed-loop eigenvectors and eigenvalues consistent, they proposed that the minimum number of constraints should be imposed to the eigenvectors' elements to limit the control effort to a reasonable amount.

The earlier studies on designing the flight control laws using eigenstructure assignment methods can be found in [8-13]. A more recent works in this area is the research performed by Wilson et al. [14]. They used eigenstructure assignment for designing a control law by constrained minimization of the difference between the desirable and achievable eigenvectors. The constraint consisted of the Lyapunov equation that was added to the minimization process. The method was applied to a mode-decoupling roll-yaw autopilot. Sobel et al. [15] improved their own method that had been proposed earlier using eigenstructure assignment by presenting a methodology for design of an advanced flight control. They applied the method for designing a flight controller for F-18 High Angle of Attack Research Vehicle (HARV) aircraft and showed the applicability of the method for constant output feedback, constrained output feedback, and output feedback with dynamic compensation for both continuous and discrete systems. Choi [16, 17] proposed an eigenstructure assignment for simultaneously assigning the right and left eigenstructures, based on the assumption that the ability of disturbance suppression in a controller is related to the left eigenstructures, and the disturbance decoupling ability is related to the right eigenstructure. The method was used to design an L-1011 flight control. A feedback control design method based on eigenstructure assignment technique has been proposed by Magni [18]. This method allowed the designer to achieve the robustness against the system parameters' variations. This control law used structured and scheduled gains. A method for designing a control law was introduced by Oliva et al. [19] that could keep following a reference attitude and maintain attitude decoupling due to the vehicle maneuvers. They designed the control law for a satellite launcher using eigenstructure assignment and optimal control. They applied their method to a linearized coupled model from a nonlinear model of the vehicle and proposed a method that was able to keep the vehicle tracking a reference attitude and decouple the yaw motion from roll and pitch. An intelligent optimization for eigenstructure assignment using neural network was introduced by Fan et al. [20]. This method minimizes the error between desired and achievable eigenvectors and was used for designing a flight control law for an aircraft. An eigenstructure assignment method has been proposed by Seo et al. considered the probability of the instability in linear time-invariant systems. This method used the probability distribution of the closed-loop eigenvalues to determine the probability of instability of those systems using the Monte Carlo evaluations. The proposed method has been used for designing a flight control [21].

The paper is organized as follow. In section 2, the mathematical basis of the orthogonal eigenstructure control is explained. In section 3, the method has been applied to design a flight control for lateral directional dynamics of an F-18 HARV at Mach 0.38 and altitude of 5000 feet. The model of this vehicle has been first addressed in [15], as an example for investigating the flight controller proposed in that paper. Finally, the advantages of orthogonal eigenstructure control for designing a flight control law are summarized in the conclusion section.

2. ORTHOGONAL EIGENSTRUCTURE CONTROL

Consider a closed-loop multi-input multi-output time-invariant linear system

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\} \tag{1}$$

$$\{y\} = [C]\{x\} \tag{2}$$

$$\{u\} = [K]\{y\} \tag{3}$$

$\{x\}$ is the $n \times 1$ state vector, $[B]$ is $n \times p$ input matrix, where $p \geq 2$ is the number of the actuators, $\{u\}$ is the $p \times 1$ input vector, $\{y\}$ is the $m \times 1$ output vector, $[C]$ is $m \times n$ output matrix, and $[K]$ is $p \times m$ feedback gain matrix. The techniques for orthogonal eigenstructure control with non-collocated actuators and sensors, or different number of actuators and sensors have been documented in [22] by the authors. This study applies the technique in [22] to a new set of problem, namely aircraft flight control design.

The method in [22] suggests adding dummy components – in the example discussed later in this paper, one dummy actuator – for calculating the gain matrix. The rows of the control gain matrix corresponding to those dummy actuators are set to zero. It is shown in [22] that the closed-loop eigenvectors belong to the achievable eigenvector set. The control input matrix B is modified to accommodate a dummy actuator.

The combined closed-loop equation of motion is

$$\{\dot{x}\} = [A + BKC]\{x\} \tag{4}$$

We define ϕ_i as the closed-loop eigenvectors, λ_i as the operating eigenvalues, and I as the $n \times n$ identity matrix. The eigenvalue problem can be defined as follow

$$[A - \lambda_i I \quad | \, B]\begin{Bmatrix} \phi_i \\ KC\phi_i \end{Bmatrix} = 0 \quad i = 1, \ldots, 2n \tag{5}$$

Since this product is zero, $\begin{bmatrix} \phi_i^T & (KC\phi_i)^T \end{bmatrix}^T$ belongs to the null space of the matrix $S_{\lambda i} = [A - \lambda_i I \quad | \, B]_{n \times (n+m)}$. Singular value decomposition of $S_{\lambda i}$ yields

$$S_{\lambda i} = [U_i]_{n \times n} [\Sigma_i \, | \, 0_{n \times m}]_{n \times (n+m)} [V_i^*]_{(n+m) \times (n+m)} \tag{6}$$

$[U_i]$ and $[V_i]$ are the left and right orthonormal matrices respectively, and $[V_i^*]$ is the conjugate transpose of the complex matrix $[V_i]$. Partitioning $[V_i]$ gives:

$$[V_i]_{(n+m) \times (n+m)} = \begin{bmatrix} [V_{11}^i]_{n \times n} & [V_{12}^i]_{n \times m} \\ [V_{21}^i]_{m \times n} & [V_{22}^i]_{m \times m} \end{bmatrix} \tag{7}$$

Achievable eigenvector ϕ_i^a of the closed-loop system is any linear combination of m columns of $[V_{12}^i]$ using an appropriate coefficient vector r^i.

$$\phi_i^a = [V_{12}^i]\{r^i\} \tag{8}$$

From equation (5) and (8), the control gain matrix $[K]$ can be determined

$$KC\phi_i^a = [V_{22}^i]\{r^i\} \tag{9}$$

To find the appropriate r^i, we define the modal energy corresponding to the i th achievable eigenvector of the closed-loop system

$$E_i = r^{i*}[V]^*[V_{12}^i]r^i \tag{10}$$

It can be seen that there is a Hermitian matrix $[V_{12}^i]^*[V_{12}^i]$ in the definition of the modal energy. Since $[V_{12}^i]^*[V_{12}^i]$ and $[V_{22}^i]^*[V_{22}^i]$ are Hermitian matrices, we can write their eigenvalue decompositions as

$$[V_{12}^i]^*{}_{n \times m}[V_{12}^i]_{n \times m} = \bar{U}^i \Lambda^i \bar{U}^{i*} \tag{11}$$

$$[V_{22}^i]^*_{m \times m}[V_{22}^i]_{m \times m} = \bar{U}_w^i \bar{\Lambda}_w^i \bar{U}_w^{i*} \tag{12}$$

$\bar{\Lambda}_i$ and $\bar{U}^i$ are the eigenvalues and eigenvectors matrices of $[V_{12}^i]^*[V_{12}^i]$, and $\bar{\Lambda}_w^i$ and $\bar{U}_w^i$ are the eigenvalue and eigenvector matrices of $[V_{22}^i]^*[V_{22}^i]$,respectively. It has been shown by the authors in [23, 24] that

$$\bar{\Lambda}_w^i + \bar{\Lambda}^i = I \tag{13}$$

$$\bar{U}^i = \bar{U}_w^i \tag{14}$$

and the eigenvalues of the Hermitian products $[V_{12}^i]^*[V_{12}^i]$ and $[V_{22}^i]^*[V_{22}^i]$ belong to the interval $[0 \;\; 1]$. Re-arranging the equation (11), we may write

$$\bar{U}^{i*}[V_{12}^i]^*[V_{12}^i]\bar{U}^i = \bar{\Lambda}^i \tag{15}$$

If the eigenvector $\bar{U}_J^i$ associated with a unity eigenvalue of $[V_{12}^i]^*[V_{12}^i]$ in equation (11) is considered as r^i, its modal energy $E^i = 1$.

$$\bar{U}_J^{i*}[V_{12}^i]^*[V_{12}^i]\bar{U}_J^i = 1 \tag{16}$$

and substituting the same eigenvector in equation (13), we have

$$\bar{U}_J^{i*}([V_{22}^i]^*[V_{22}^i])\bar{U}_J^i = 0 \tag{17}$$

From equations (16) and (17), we may conclude

$$[V_{22}^i]\bar{U}_J^i = 0 \tag{18}$$

That implies the gain matrix is zero

$$KC\phi_i^a = [V_{22}^i]r^i = [V_{22}^i]\bar{U}_J^i = 0 \tag{19}$$

It shows that the open-loop system has been regenerated. Therefore, if the eigenvector $\bar{U}_J^i$ associated with the unity eigenvalue of $[V_{12}^i]^*[V_{12}^i]$ is selected as r^i, the open-loop eigenvectors within the null space of the closed-loop eigenvectors associated with the operating eigenvalue λ_i is regenerated. The open-loop eigenvectors are the intersections of the open-loop eigenvector set and the achievable eigenvector set. Other eigenvectors associated with the non-unity eigenvalues of $[V_{12}^i]^*[V_{12}^i]$ are orthogonal to the eigenvector associated with the unity eigenvalue. Therefore, a set of closed-loop eigenvectors can be found, orthogonal to the open-loop ones. This concept is shown in Fig. 1.

The orthogonal eigenstructure control finds one open-loop eigenvector for each operating eigenvalues, and simultaneously finds $m-1$ vectors within the achievable eigenvectors set orthogonal to the open-loop eigenvector. Depending on the number of actuators, one can repeat this process for m operating eigenvalues. As a result, there are m^m closed-loop systems that a controller designer has to choose one of them with the most desirable performance. Since a regenerated open-loop system is included in the produced closed-loop systems with a zero gain matrix, the actual number of the closed-loop systems is $m^m - 1$.

The calculated closed-loop eigenvectors are appended for all the operating eigenvalues

$$V = \left[[V_{12}^1]r^1 \cdots [V_{12}^m]r^m \right] \tag{20}$$

$$W = \left[[V_{22}^1]r^1 \cdots [V_{22}^m]r^m \right] \tag{21}$$

The control gain matrix K that includes dummy actuators is

$$K = W(CV)^{-1} \tag{22}$$

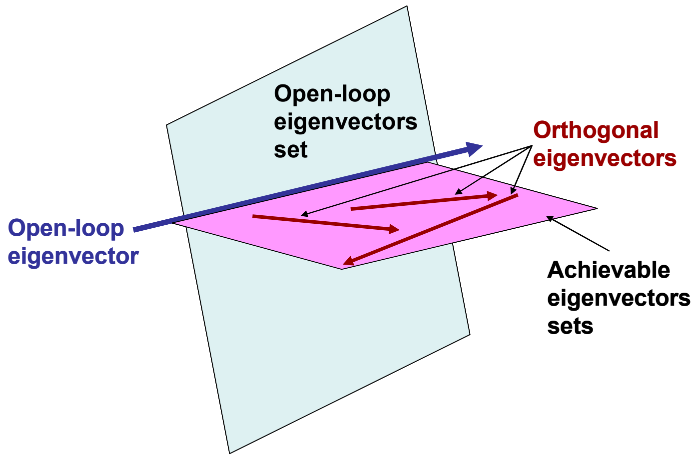

Fig. 1. Schematics of the process of orthogonal eigenstructure control. For each operating eigenvalue, the control method finds several vectors orthogonal to the open-loop eigenvector. The orthogonal vectors are within the achievable eigenvectors set. The open-loop eigenvector will be replaced by one of the calculated orthogonal vectors.

The rows of the control gain matrix associated with the dummy actuators are set to zero. Using this reduced gain matrix and the original $[B]$ matrix, we have the state matrix of the closed-loop system as follow

$$A_c = A + BKC \tag{23}$$

3. EXAMPLE

As an example, we use lateral directional dynamics of an F-I8 HARV aircraft linearized about an operating condition that represent a speed of 0.38 Mach, altitude of 5000 ft, and angle of attack of 5°. This problem has been modeled and used by Sobel et al. in [15]. First-order actuators and a yaw rate washout filter are augmented to the aerodynamic model of the vehicle. The state matrix A, input matrix B, and output matrix C are reported as follow:

$$A = \begin{bmatrix} -30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -30 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -30 & 0 & 0 & 0 & 0 & 0 \\ -0.0070 & -0.0140 & 0.0412 & -0.1727 & 0.0873 & -0.9946 & 0.0760 & 0 \\ 15.3225 & 12.0601 & 2.2022 & -11.0723 & -2.1912 & 0.7096 & 0 & 0 \\ -0.3264 & 0.2041 & -1.3524 & 2.1137 & -0.0086 & -0.1399 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0.0875 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5000 & 0 & -0.5000 \end{bmatrix}$$

$$B = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 30 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The state vector for this model, as described in equation (1) is $X = \left[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\right]^T$. x_1 is aileron deflection, x_2 is stabilator deflection, x_3 is the rudder deflection, x_4 is the sideslip angle, x_5 is the roll rate, x_6 is yaw rate, x_7 is bank angle, and x_8 is the washout filter state. The three control commands are aileron command u_1, stabilator command u_2, and rudder command u_3. The outputs are washed out yaw rate y_1, roll rate y_2, sideslip angle y_3, and bank angle y_4. As it can be seen, the system includes 3 inputs and 4 outputs. As stated earlier, using the method proposed in [22], we add a dummy actuator to the system. Therefore, a column is added to the input matrix B, with zero elements, except the element that is associated with the direct input of the dummy actuator itself. The value of this element is arbitrary, and is considered to be 30, for convenient.

As a result, the new input matrix B becomes

$$B = \begin{bmatrix} 30 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 \\ 0 & 0 & 30 & 0 \\ 0 & 0 & 0 & 30 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The locations of the open-loop and closed-loop systems are shown on Fig. 2. A significant move in the locations of the poles can be observed. Also, the time histories of the outputs of the open-loop and closed-loop systems are depicted on Fig. 3. The simulation has been run for a 1° change in the angle of attack as an initial condition. It is observed that the angle of attack decreases to zero and also the couplings between the angle of attack and other states are significantly reduced. Comparing the results to the results of ref. [15], we see the states are satisfactorily decoupled such that excessive overshoots are prevented as stated in Table 1. Control commands are shown on Fig. 4. It shows that the overshoots in the time histories of the control commands are significantly smaller than the result in [15]. The maximum overshoots of the outputs and the control commands are shown in Table 1.

Table 1. Maximum absolute value of the outputs and control commands overshoots

	roll rate	yaw rate	sideslip angle	bank angle	aileron command	stabilator command	rudder command
Current results	0.1003	0.2571	0.0187	0.0376	0.5853	0.0590	1.3023
Ref [15] results*	1.5	0.2819	0.2	0.0532	0.25	1.4	3.4

* The approximated results are according to Figs. 2 and 3 of Ref [15].

The roll rate and sideslip angle have been reduced 15 and 10 times respectively. Also 9% and 29% reduction in the maximum yaw rate and bank angle can be seen. In addition to the reduction in the maximum amount of the aforementioned parameters, the maximum actuation forces are reduced significantly. Comparing the result of the orthogonal eigenstructure control to the result of the eigenstructure assignment method reported in [15], The stabilator and rudder commands are reduced 29 and 2.6 times respectively. As a trade off, however; the maximum of aileron command is increased 2.3 times. It can be concluded that the application of orthogonal eigenstructure control results in more desirable mode decoupling with lesser actuation forces.

4. CONCLUSIONS

Orthogonal eigenstructure control has been used to design a flight control law. It was shown that this method successfully decoupled the dynamic modes of the vehicles, without the need of any input from designer, such as desirable locations for closed-loop poles or desirable shapes of the closed-loop eigenvectors. The method automatically generated a class of all closed-loop systems. The flight control law was determined for the lateral directional dynamics of an F-18 HARV and the results were compared to the results of an eigenstructure assignment method in an earlier study. The results showed that a great mode decoupling could be achieved while the actuation forces are significantly reduced.

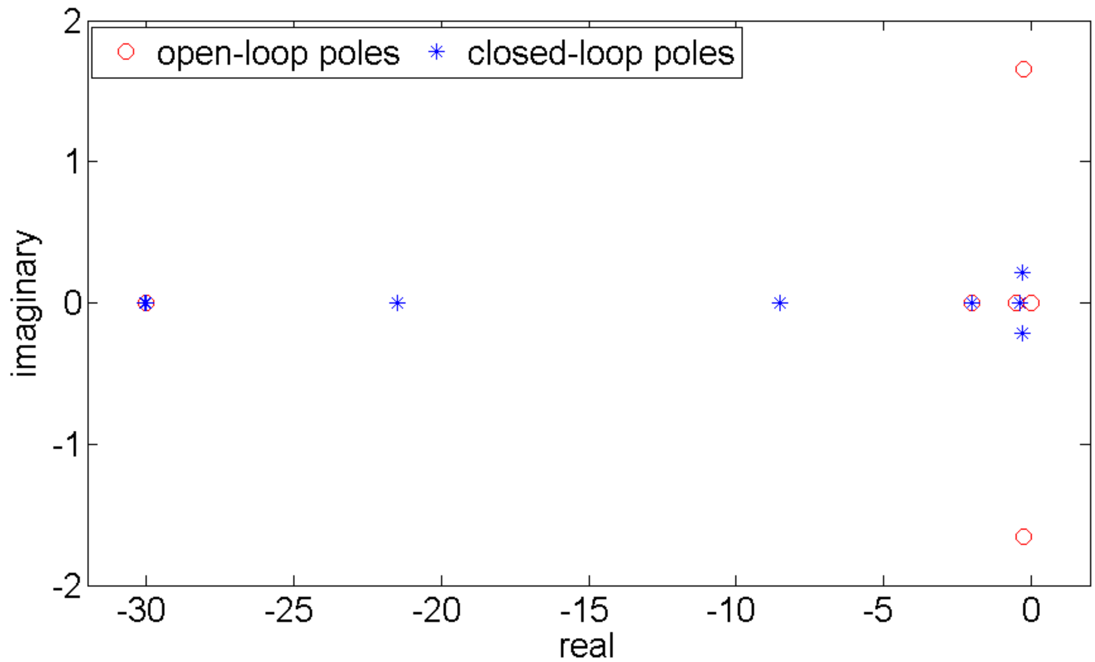

Fig. 2. open-loop and closed-loop poles

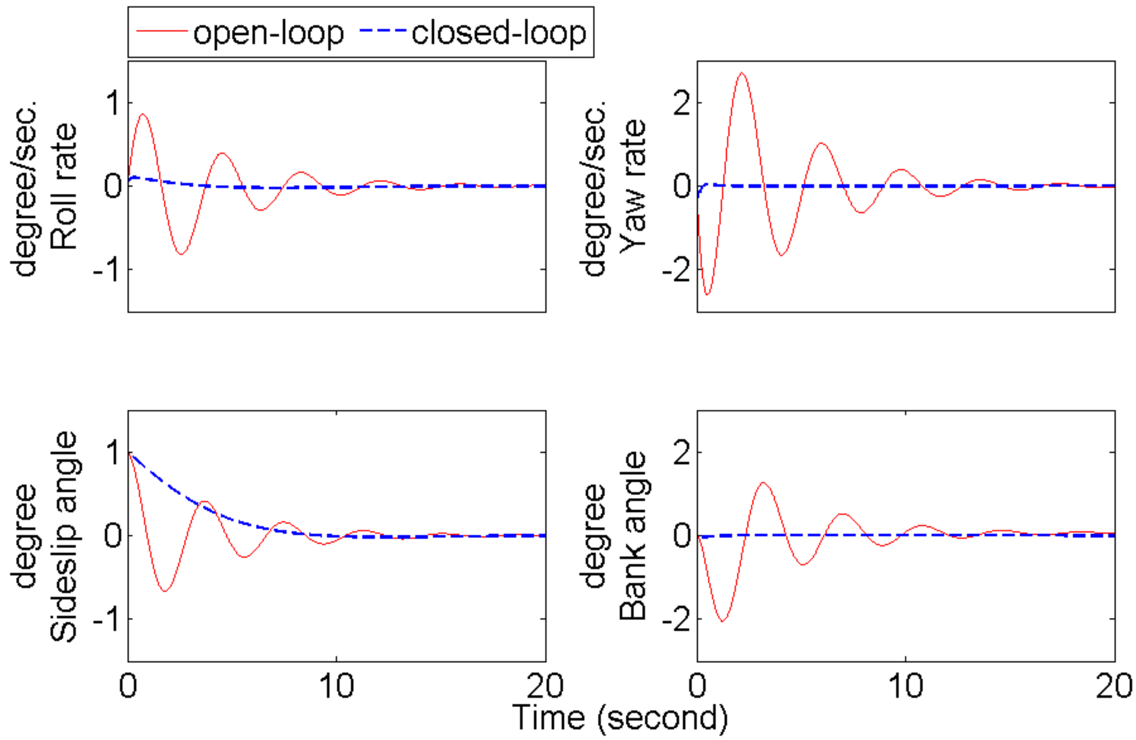

Fig. 3. Time histories of the outputs

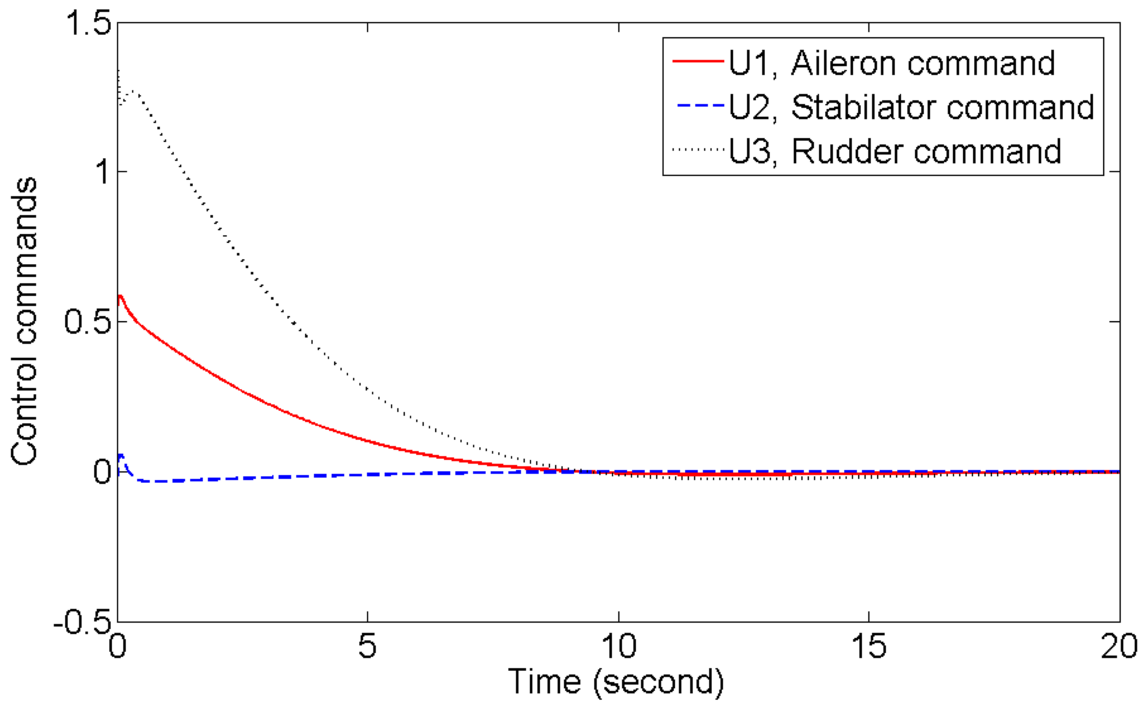

Fig. 4. Time histories of the control commands

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