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A review on eigenstructure assignment methods and orthogonal eigenstructure control of structural vibrations

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Abstract. This paper provides a state-of-the-art review of eigenstructure assignment methods for vibration cancellation. Eigenstructure assignment techniques have been widely used during the past three decades for vibration suppression in structures, especially in large space structures. These methods work similar to mode localization in which global vibrations are managed such that they remain localized within the structure. Such localization would help reducing vibrations more effectively than other methods of vibration cancellation, by virtue of confining the vibrations close to the source of disturbance. The common objective of different methods of eigenstructure assignment is to provide controller design freedom beyond pole placement, and define appropriate shapes for the eigenvectors of the systems. These methods; however, offer a large and complex design space of options that can often overwhelm the control designer. Recent developments in orthogonal eigenstructure control offers a significant simplification of the design task while allowing some experience-based design freedom. The majority of the papers from the past three decades in structural vibration cancellation using eigenstructure assignment methods are reviewed, along with recent studies that introduce new developments in eigenstructure assignment techniques.

1. Introduction

Eigenstructure assignment methods have been used as efficient methods of vibration suppression during the past three decades. These methods allow unbounded design freedom to the designers beyond the simple pole placement. The design freedom can be used by defining a proper shape for modes of structures. While the eigenstructure assignment methods offer a large and complex design space of options, recent development of orthogonal eigenstructure control offers a significant simplification of the design task. In this paper, different eigenstructure assignment methods are reviewed with an emphasis on those that are used for structural vibration suppression.

2. Eigenstructure assignment

Eigenstructure assignment, whether state feedback or output feedback, has an important role in active control of the transient responses in Multi-Input Multi-Output (MIMO) systems to either improve the performance of the closed-loop system, or minimize the required control effort. For example, when a stabilizable system has uncontrollable modes, state feedback control cannot change the associated eigenvalues, but the associated eigenvectors can be shaped in order to have a more smooth response [1].

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Wonham [2] studied eigenvalue placement for MIMO systems and proved that this property of state feedback is applicable to controllable systems. Later, it was Moore [3] who introduced eigenstructure assignment and identified the set of possible eigenvectors associated with a distinct set of closed-loop eigenvalues. He showed that there is a considerable design freedom beyond the simple pole placement that allows the controller designer to both place the eigenvalues and assign the related eigenvectors; thus, both the speed of response that is determined by the closed-loop eigenvalues, and the shape of response that is related to the closed-loop eigenvectors can be controlled. Klein and Moore [4] also presented an algorithm for non-distinct closed-loop eigenvalues and their related eigenvectors keeping the full state feedback scheme by calculating the basis of the null space of the combined system.

A parametric approach introduced by Fahmy et al. [5,6] using the determinant of the combined system matrices. This method identifies the class of achievable eigenvectors and describes explicitly the generalized eigenvectors associated with the assigned eigenvalues. Their approaches to eigenstructure assignment are similar to [3,4]; however, instead of taking an ad hoc approach by identifying the appropriate closed-loop eigenvectors, the design freedom apperas in terms of choosing the appropriate free parameters in the process of finding the state feedback gain matrix. This leads to extra free parameters if the entire open-loop eigenvalues are not required to be moved [7].

The aforementioned full state feedback eigenstructure assignment methods do not suggest an alternative solution if the control gain matrix does not lead to a desirable closed-loop system [8]. Additionally, output feedback control is more desirable from a practical point of view. One of the early research on the eigenstructure assignment with output feedback approach was performed by Srinathkumar [9]. He derived sufficient conditions to assign a set of arbitrary distinct eigenvalues and showed the maximum number of assignable eigenvalues for the controllable systems. He also determined the maximum number of eigenvectors and the elements in each eigenvectors that can be chosen arbitrarily based on the number of inputs and outputs.

Cunningham [10] used Singular Value Decomposition (SVD) for finding the basis of the null space for achievable eigenvector subspace. A finite number of actuators are needed to shape the eigenvectors of the MIMO system utilizing SVD [11]. This was the first practical method of eigenstructure assignment in order to have a desirable transient response behavior. In this output feedback control method, the basis vectors were optimally combined to either minimize the error between achievable and desirable eigenvectors or to assign the exact eigenvectors.

Andry et al. [8] studied full state, output, and constrained output feedback to shape the transient responses of a dynamical system. For partial output feedback, they set some of the elements of the feedback gain matrix to zero to reduce the controller complexity. A knowledge of the behavior of the system is still required when their method is utilized, since the stability is not mathematically guaranteed. Calvo-Ramon [12] defined a partial output feedback eigenstructure method to select the zeros in gain matrix to place the desired eigenvalues with a simpler feedback. He proposed an algorithm based on minimizing the distance between the desired eigenvectors and achievable eigenvectors considering the fact that desired eigenvectors do not usually lie in the corresponding achievable subspaces.

2.1. Normal mode localization and vibration cancellation

Normal mode localization is a phenomenon that a structure divides into two areas of small and large amplitude of vibration. It happens when one or more global modes of vibration become confined to a local region of the structure. Some disorders in the periodicity of a structure, such as material discontinuity, create mode localization. It means that the energy of vibration coming to the structure is not able to propagate far through the structure and will be confined to the areas close to the source of vibration [13].

Material discontinuity, in classical periodic isolator structures, creates stop bands that suppress the wave propagation of external disturbances within a particular frequency range. Hodges [14] showed that the normal mode localization mostly occurs in the structures with large number of weakly coupled substructures. The steady-state responses of these structures decay exponentially away from the vibration source. In other words, disruption of the periodicity causes the attenuation of vibrational waves in all frequency bands irrespective of dissipations in the system. In mode localization, eigenvectors have the major influence in distributing vibrational energy. Localization occurs when the eigenvalues are clustered in a small band. The localization degree depends on the ratio of disorder to coupling, with a stronger localization when this ratio increases. Localized modes in disordered structures can be useful or harmful. For example, localization can be harmful if it results in fatigue as it happens in turbine blades.

Displacements are relatively large in the area of localization, with localized amplitudes frequently ten times that of the isolated areas of the structure [15]. Also, localization can affect the robustness and stability of active control systems for large space structures if the control law is designed assuming a large numbers of modes of the structure. For the class of large space structures; however, localization can be used as a method of vibration confinement at the areas close to the source of disturbance [16].

Vibration confinement is the manipulation of the structural vibrational modes in order to reduce the amplitude of vibrations in the concerned areas of the system. The energy of vibration is confined to the remaining parts of the system which are less important or their isolation is much easier in practical sense than the isolated areas. It has been shown that methods based on vibration confinement reduce the amplitude of vibration much more effectively than other methods of vibration suppression [17,18]. Most of the active control methods emphasize on damping or cancellation of disturbance; therefore, they have to be designed for specific disturbances or inputs. The actuators provide sufficient damping for energy dissipation to reduce vibration in all parts of the structure almost in the same amount. In other words, those algorithms decay the vibration at both the isolated and the confined part at the same rate which does not necessarily comply with the performance specifications [11]. The conventional methods are considered to have global objectives in comparison to eigenstructure assignment that has local effects on the structural response [17]. Using the concept of the normal mode localization phenomenon in periodic structures, Song et al. [18] proposed an algorithm for choosing a linearly independent set of mode shapes in order to reduce the relative vibrational amplitude at certain areas by redistributing the vibrational energy. This algorithm could be applied to any system that shows an oscillatory behavior. Near orthogonality is a desirable condition to have insensitive eigenvalue placement to modeling uncertainty. This means that corresponding modal matrix has the minimum condition number. A minimum condition number in turn means spillover suppression [19].

Rew et al. [20] proposed a non-iterative algorithm based on QR decomposition or SVD methods, applicable to both state and output feedback, while the design freedom is used for minimizing the gain. This method projects the closed-loop eigenvectors onto the achievable eigenvector subspace. Closed-loop eigenvectors are close to the desired one in a least square sense. Since arbitrary selection of acceptable eigenvectors may cause an ill-condition eigenvector modal matrix, closeness of the closed-loop eigenvectors to the achievable ones becomes important. If the closed-loop and the achievable eigenvectors are not close, the associated gain matrix cannot be accurately calculated and the closed-loop system may be highly sensitive. In the latter reference, the algorithm determines the closed-loop eigenvectors close to both desired and open-loop ones in a least square sense. One problem that occurs in large flexible structures (especially space structures) is that they have a large number of lightly damped modes that are very close together. Use of a high authority controller can cause spillover- induced instabilities. Low-authority controllers in conjunction with high authority ones makes the stability of the system robust. This class of controllers uses collocated sensors and actuators and is called static dissipative controllers. This method is a sequential method to calculate velocity and position gain matrices to assign an arbitrary number of closed-loop eigenvalues to their specific locations and has been studied and applied by Maghami et al. [21,22]. While partial eigensystem assignment with output feedback for passive linear time-invariant systems can make a closed-loop system unstable, output feedback, with dissipative controllers such as flexible space structures lead to stable systems.

Shelly et al. [15] studied the absolute displacement in the first and second order systems, since it is not possible to predicts how the absolute displacements in a system change merely by changing the system's eigenvectors. They introduced eigenvector scaling as a mode localization technique for shaping time domain response of the system. This method adjusts specific parts of each eigenvector to uniformly increase the relative displacement of the corresponding areas in the system [23]. They showed analytically that absolute displacements in isolated areas can be reduced by eigenvector shaping, regardless of the type of the disturbance. The experimental result has been shown in [24]. The test rig is a simply-supported beam subjected to an impulse disturbance and Eigenstructure Realization Algorithm (ERA) was used to model the system. In other studies by the same group of researchers, a numerical simulation of vibration confinement in a pinned-pinned beam was performed [25,26] such that two piezoelectric actuators and sensors were used to confine the vibrations within a simply supported beam. Both passive and active control methods were used. The passive control reduces the number of required actuators and also reduces their operating power which leads to a lower application cost. This eigenstructure shaping method for active control of the beams is called eigenvector scaling. This method regenerates the behavior of the system by scaling and reforming the system mode shapes partly or entirely when passive mode localization happens. Vibration confinement of the system is not sensitive to the type of disturbance because all the mode shapes are scaled in the same way. An application of this method [17] that experimentally utilizes this feedback control method for vibration suppression shows that the scaling matrix magnifies the coupling force applied by the isolated area and reduces the coupling force applied by the localized area. Therefore, the feedback control makes simultaneous stiffening and softening of the coupling between the isolated and localized regions. The consistent scaling of the eigenvectors makes the relative displacement of isolated and localized areas constant over the entire range of the eigenvalues. On the contrary, passive localization, as occurs in some structures due to irregularities, makes confined areas move with excitation frequency. As a result, for any disturbances applied to the isolated area, the absolute displacement in localized area will increase but the displacement in the isolated area will remain almost intact. Also, if a disturbance applies to the localized area, the absolute displacement in the isolated areas will decrease but the localized area will remain the same [17]. Uniform scaling has a drawback because the system has to have pairs of actuators and sensors equal to the number of coupled modes to control the interaction between the neighboring systems. For example, when adjacent masses interconnected with spring and damper the maximum and also optimum number of the pairs of actuators and sensors is two. The number of masses is not a parameter for the number of pairs of actuators and sensors. Therefore, not only the number of required pairs of actuators and sensors for continuous systems is infinite but also a large number of actuators and sensors are needed for their linearly discretized models. SVD-eigenvector shaping, a modification of the former method, introduced as a solution to the problem of limited pairs of actuators and sensors [27]. This method is a combination of authors' earlier method and Cunningham [10] approach. SVD-eigenvector shaping uses Moore-Penrose generalized left inverse and produces the closest eigenvector in least square sense to the desired ones, since it minimizes Euclidean 2-norm error. The result of applying this method to a simply supported beam, as a distributed parameter system, is presented in [27].

Tang et al. [28,29] used an active-passive hybrid vibration confinement system using a network of piezoelectric actuators as active elements and inductors and resistors as the passive elements of the systems. Rayleigh principle was used for finding the optimal eigenvectors to eliminate the necessity of pre-selecting the closed-loop eigenvectors. After finding the subspace for the eigenvectors, one can determine the optimal achievable eigenvectors that have the minimal vector 2-norm in concerned regions that eliminates the problem of closeness of the desired and achievable eigenvectors. On the other hand, a perfect match between the desired eigenvectors and the achievable one is not necessary, because the vibration suppression is concerned in certain degrees of freedom. Pre-determination of the desired eigenvector components can cause unsatisfactory performance if a match between components of the desired and achievable eigenvectors in the unimportant degrees of freedom happens. This makes a trade-off on the closeness of the important degrees of freedom. Since the passive circuitry elements are used as extra design variables, it's model has large degrees of freedom. Also the passive electrical elements can be assigned to be the confined part of the hybrid system. Obviously, the change in electrical elements is much easier than changing the mechanical structure or elements. A case study of this method is presented in [30].

Eigenstructure assignment with distributed feedback, has been proposed by Choura and Yigit [31,32] for one and two dimensional structures such as uniform strings and beams. The feedback control reduces the settling time of some parts of the system while increases the settling time at the rest of the system by altering the mode shapes to exponentially decaying functions of space. This method can be defined an inverse eigenvalue problem for optimization of the system such that the most desirable geometry and material properties for confining the vibrational energy can be determined. The drawbacks of this method are the facts that the control method is a full state feedback, and the mass, damping, and stiffness matrices for the closed-loop system are highly populated. To address the problem of limited actuators, partitioning the system into two subsystems are considered by Choura and Yigit in [11]. This method still requires full state feedback in the form of acceleration feedback; however, the states that are not being measured need to be estimated by a dynamic observer in order to meet practical issues. A case study on the axial vibration confinement of the rods using this method is presented in [33]. A similar method for linear time varying systems is reported in [34]. Even though from a theoretical stand point, the proposed method is novel, the need for full state feedback reduces the applicability of the methods to the systems with large degrees of freedom.

2.2. Robustness in eigenstructure assignment

Robustness in eigenstructure assignment is the degree of insensitivity of the closed-loop eigenvalues to the uncertainties and perturbations in the system. Satisfying different stability robustness criteria as well as performance robustness criteria are the main subjects of robust control. Among different control methods, eigenstructure assignment is a simple and efficient method. One of the first studies on the robustness of eigenstructure assignment methods was performed by Kautsky et al. [35]. They determined robust and well-conditioned solutions for state feedback control problems. These methods minimize the feedback matrix norm upper bound and maximize the lower bound of the stability margin. Slater et al. [36] presented an eigenstructure assignment method for constrained state or output feedback by considering the problem of movement of neighborhood of the closed-loop eigenvalues. They explained an important observation that a large change in eigenvectors may need a large movement of the eigenvalues in order to minimize the feedback control gains. Therefore, if the changes are applied merely to the eigenvectors, the control efforts will not be necessarily minimized when the closed-loop eigenvalues are kept close to the open-loop eigenvalues. As a result, in order to avoid the large control efforts, closed-loop eigenvalues and eigenvectors have to be consistent and a minimum number of constraints should be imposed to the eigenvectors' elements.

Mudge and Patton [37] presented the procedures for both modal decoupling and robust eigenstructure assignment when the allowable subspaces have been determined using SVD. They presented a modally shaped design application when the desired modal structure of the closed-loop system response is known. For the cases when the desired forms are not achievable, since they do not lie in the allowable subspace, they may be projected in a least-squares sense into the required subspace. Also, they proposed a robust eigenstructure design based on Kautsky [35] method when no prior knowledge of the modal coupling is available and there are no criteria to choose desirable eigenvectors.

Sobel et al. [38] determined the sufficient condition for stability robustness of linear systems with time-varying norm bounded structural uncertainty. They showed that the nominal eigenvalues have to lie to the left of a vertical line in the complex plane that is determined by a norm involving the structure of the uncertainty and the nominal closed-loop eigenvector matrix. Using this result, a robust eigenstructure assignment design method was proposed by Yu et al. [39]. This approach optimizes either the performance robustness or sufficient condition for stability and constrains the dominant eigenvalues to lie within chosen regions in the complex plane. Pre-specified eigenvalues have a restrictive effect on the eigenvectors and diminish the domain in which eigenvectors can reside. On the other hand, pre-specified eigenvectors have restrictive effect on the pole placement. It is known that the closed-loop poles have to lie in certain areas in order to have a stable system. Another algorithm that considered the proper location for the closed-loop eigenvalues was proposed by Keel et al. [40]. They determined a robust state feedback to restrict the closed-loop eigenvalues' locations within an arbitrary neighborhood around their desired locations. The bounded parameter perturbations of the open-loop matrix will not move the closed-loop eigenvalues too far from their desired locations.

Distinction between right and left eigenstructures have also been used in several research work to address the robustness of eigenstructure assignment. Kwon et al. [41] generalized a simple eigenstructure assignment via output feedback in linear multivariable systems using the left and right eigenvectors. Their method does not need the assumptions that eigenvalues of the closed-loop system are distinct or different from eigenvalues of the open-loop system. Even though the eigenstructure assignment techniques for vibration cancellation generally use the right eigenvectors, in some studies the left eigenvector assignment was employed. Choi et al. [42] presented a general left eigenstructure assignment method, based on the bi-orthogonality condition between the right and left modal matrices of a system. This method considers both the modal disturbance suppressibility and modal controllability in control systems design. A more general work was presented in [43].

2.3. Eigenstructure assignment for second order systems

To describe the dynamics of a structural system, usually a second-order differential equation is used, with structural matrices that are symmetric and sparse. On the other hand, control theory and estimation techniques are established for first order realization of the systems. Therefore, a large effort is made to transfer second order equations to first order configuration. The first order system of equations increases the dimension of the equations and loses the symmetry and sparsity of the structural matrices. Thus, a model reduction is needed before applying a controller or an estimator to the system in order to reduce the difficulty of the numerical computations. An output feedback method based on SVD or QR decomposition was proposed by Maghami et al. [44]. The coefficients corresponding to the basis of the null space are computed using an algorithm based on subspace intersections, after finding the orthogonal null space. Juang et al. [45] proposed a method that designs an estimator and a controller for the second order system. Also, a controller design without estimator is reported by Juang et al. [46] that can be applied to both state and output feedback control. Because the system is second order, displacement and velocity or velocity and acceleration can be used to assign the eigenvalues. This is interesting because of the practical implementation and frequent use of accelerometer in practice. The closeness of the closed-loop eigenvectors to both open-loop and desired one is the objective of this algorithm which is the result of the intersection of the achievable and desirable eigenvectors. The importance of this method is to keep the computational features of a second order system, such as symmetry and sparsity of the mass and stiffness matrices. SVD is used to find the achievable eigenvector subspace in this method. Chan et al. [47] solved a similar problem by combined derivative and proportional state feedback for almost arbitrary eigenvalue assignment.

There are several other interesting approaches for eigenstructure assignment in second order systems. Triller et al. [48] applied eigenstructure assignment in a new control coordinate system based on the substructure representation of the system. This coordinated system is able to accurately predict the actuators' forces while a model reduction has been applied. Datta et al. [49] developed an algorithm for the partial eigenstructure assignment of quadratic matrix pencil. Their scheme directly determines the control gain for second order mechanical systems with no need to transform them to the state-space form. Another technique for eigenstructure assignment to an actively controlled second order system was presented by Schulz et al. [50]. This method is a parametric method and optimizes the controller and structural parameters. The feedback gains are explicitly parameterized in terms of the eigensystem and structural matrices. The closed-loop system is optimized by minimizing the norms related to the feedback gains. Partial pole assignment in vibratory systems while the other poles of the system are kept unchanged was studied by Ram et al. [51]. This non-iterative method is applicable to systems with second order differential equations. This method leads to a unique solution for single input single-output problems but produce a family of solutions for MIMO control problems.

2.4. Combination of eigenstructure assignment methods and other conventional control methods

In order to have a robust control method and reach optimality, many researchers used eigenstructure assignment and optimal control methods together. Some of these work have been discussed earlier in this paper. Also, there are some work that use Sylvester equation directly for optimal eigenstructure assignment in linear systems or general descriptor systems that we review in this section.

Maghami [52] used dissipative output feedback gain matrices for assigning a selected number of closed-loop poles. The method is a sequential procedure that assigns one self-conjugate pair of closed-loop eigenvalues at each step using dissipative output feedback gain matrices. The algorithm checks the eigenvalues assigned in the previous steps not to be disturbing the system. It is important to notice that the previously assigned eigenvalues may become unstable when new pair of eigenvalues are being assigned. The constraint of being different from zero on the closed-loop eigenvalues was removed for state feedback control of the descriptor system in [53]. This work uses the parametric solution of generalized Sylvester matrix equation to apply an output feedback to a descriptor system [53]. In a similar concept, output feedback control with a reduced orthogonality condition was developed by Clarke et al. [54]. This method relaxes the requirements on the number of eigenvectors that must be specified before assigning a set of eigenvalues. It consists of two stages for assignment of eigenvectors, and allows a subset of eigenvectors to be assigned before assigning the rest of the degrees of freedom to the eigenvectors in next stage. Syrmos et al. [55] proposed an independent output feedback algorithm using the solutions of two coupled Sylvester equations. A numerical state feedback approach for multi-input eigenstructure assignment for descriptive systems was given by Mimins [56].

Duan and Patton have performed extensive amount of research on eigenstructure assignment in descriptor systems. A parametric robust control design of descriptor systems using eigenstructure assignment was presented by Patton [57] utilizing both the desired closed-loop pole placement in the time domain and the minimization of a robustness index in the frequency domain are considered. Duan [58] studied robust eigenvalue assignment in descriptor linear systems via proportional plus partial derivative state feedback control. Duan [59,60] also proposed an eigenstructure assignment for multivariable linear systems using state feedback control where the set of achievable eigenvectors is calculated by SVD, and then they are parameterized to provide the entire design freedom. The same method was applied to second order linear systems using a proportional plus derivative feedback controller [67,68] and to high-order linear systems [61] and high-order descriptor linear systems [62]. The general parametric solution of the general high order Sylvester equation has been given in [63].

Duan [64] also presented parametric approach to eigenstructure assignment in multivariable linear systems by parametric design of dynamical compensators. This method treats the problem of robust dynamical compensator design, in the sense that the closed-loop eigenvalues are insensitive to small partial variations in the open-loop system matrices. Also, utilizing the state feedback controller [65] and descriptor system [66] were proposed using similar approach. Eigenstructure assignment by decentralized output feedback as parametric solution of a generalized Sylvester matrix equation for linear systems [67], continuous descriptor systems [68], and linear descriptive systems [69] are amongst his other works that emphasize on the solution of Sylvester equation [66,70].

Liu et al. [71] proposed an explicit parametric expression for gain matrix of a state feedback control. A high-gain state feedback eigenstructure assignment that is insensitive to the perturbations, has been reported by Sen et al. [72]. The perturbations are the parameter perturbations in both the plant and the gain matrices [73]. Liu et al. [74] also proposed a control algorithm that combines the time-domain performance by eigenstructure assignment and frequency domain robust performance with H_∞ specifications for a descriptor system. This method is a parametric state feedback control and is a joint optimal robust control design in both time and frequency domains [74]. Based on the same optimization method, they proposed a robust eigenstructure assignment control using genetic algorithms and gradient-based approach has been proposed [75,76].

3. Orthogonal eigenstructure control

It has been shown that eigenstructure assignment methods are effective for active vibration cancellation in structures and wherever a decoupling control is needed such as a flight control. The methods currently available however, depend on the experience of the controller designer, based on the geometry and dynamics of the structures. Existing eigenstructure assignment methods require a priori definition of the desired eigenstructure. The desirable eigenvectors; however, do not necessarily lie within the space of achievable eigenvectors. This difference between the desirable and achievable eigenvectors generates an error that may result in unsatisfying performance in the controlled system. This may lead to excessive actuation forces because of improper location of the associated closed-loop poles.

A new approach to eigenstructure assignment was proposed by the authors [77–79], motivated by the need for an active vibration cancellation method that is relatively easy to design and implement. This approach allows control engineers to achieve good performing designs by keeping the range of suitable option within the design to a manageable size.

Orthogonal eigenstructure control is an output feedback control for structural vibration cancellation. This control method automatically leads to a set of desirable closed-loop eigenvectors that result in mode decoupling and disturbance rejection. It finds the closed-loop eigenstructures within the achievable eigenvector set while they are orthogonal to the open-loop eigenvectors to the feasible extents. This control method does not need a pre-determination of the closed-loop eigenstructure; thus, a prior knowledge of the closed-loop system is not required.

The goal for orthogonalizing the eigenvectors is to find a system with decoupled modes. Decoupled modes require that the eigenvectors of the system become orthogonal to each other to the extent feasible, thereby minimizing the spillover of energy from one mode to another. While the available control design methodologies offer a large and complex design space of options that can often overwhelm a designer, this control method offers a significant simplification of the design task while still allowing some experience-based design freedom. This control method reduces those design freedoms that are not guarantied to help the disturbance rejection purposes. It reduces the design freedom to use of vectors orthogonal to the eigenvectors of the system and let the controller designer to substitute the eigenvectors with vectors that are almost orthogonal to them. Even though the design freedom is reduced, but the control method focuses the design freedom in a more efficient direction. This method is extended to be applicable to systems with non-collocated or different number of actuators and sensors [79,80]. This broadens the applicability of orthogonal eigenstructure control applicable to general systems. For example, design of a flight control law has been studied in [81].

4. Conclusions

Eigenstructure assignment methods that have been used widely during the past three decades for vibration suppression, especially in large space structures were reviewed. Different methodologies have been proposed by researchers for using the design freedom offered by eigenstructure assignment beyond the simple pole placement. Eigenstructure assignment methods can be applied to flexible structure whether the first order or second order model of the structure are available, resulting in relatively robust vibration isolation. These methods, however, offer a large and complex design space of options which can become difficult to manage during the design process. Orthogonal eigenstructure control addresses those issues and offers a significant simplification of the design task while still allowing some experience-based design freedom.

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